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ELEMENTS OF THE STRUCTURAL SYNTHESIS OF CONTROL RELAY CIRCUITS

By V. N. Roginskiy

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ELEMENTS OF THE STRUCTURAL SYNTHESIS OF CONTROL RELAY CIRCUITS

[Following is the complete translation of the book by
V. N. Roginskiy entitled "Elementy strukturnogo sintezo
releynykh skhem upravleniya" (English version above),
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With less loss of time than by the intuitive method.

The extensive application of theory to practice makes it possible to point out the shortcomings of this method and to develop both a theory and practice of the use of such methods.

The author hopes that the book will contribute to a broadening of the field of application of the theory in the practical activity of engineers and scientific research workers, and to contribute to a further development of the theory.

The author expresses his gratitude to Doctor of Technical Sciences M. A. Gavrilov and Doctor of Physical-Mathematical Sciences, S. A. Yanovskaya for many valuable indications, and also to Candidate of Technical Sciences, A. D. Kharkevich, V. G. Lazarev, G. N. Povarov, Candidate of Philosophical Sciences, T. L. Maystrova, A. A. Arkhangel'skaya and Yu. L. Sagalovich for advice and help in the development of individual problems.

INTRODUCTION

The main problem of electrical communication is to insure reliable transmission of communications over a distance from one point to another (and sometimes from one point into many points) with the aid of electrical means.

In accordance with this task, the scientific-technical problem [1, 2] can be broken up into basic groups:

1) Insure the transmission of communication over a distance.

2) Insure connections between definite points.

In the first group of problems, one solves problems of producing communication channels. The principal indices here are the correspondence between the received communication and the transmitted one, and the degree of utilization of the volume of the channel and of the communication line.

The second group of problems concerns the construction of networks, stations, and circuits of the commutation apparatus, such as to insure the possibility of interconnecting any points in the line at minimum cost, or to transmit communication in the necessary direction. The main qualitative index here is the speed of establishment of connection or the speed of transmission of communication from one end point to the other.

The problems of the second group can be presented in general form as follows. There exists an area, over which subscribers are located --- sources and receivers of information. A certain volume of information is transmitted between them, and is of random character. It is required to construct a network, i.e., to join all the subscribers with each other and with the communication channels in such a way, as to insure on the one hand the required quality of service (number of failures, waiting time, delay, etc.), and on the other hand at a minimum cost. For the sake of economy, individual groups of subscribers are connected to common stations. The connection between the latter is either direct or through intermediate station -- centers.

Within the stations and the centers there should be apparatus that connects or transmits communications in the necessary direction. This apparatus can be subdivided into switchboard apparatus and control apparatus.

Previously, switching and control systems were considered as a whole. Recently there is a separation of the apparatus of these systems because of the difference in the functions performed and the qualitative and quantitative requirements.

Switching apparatus should insure the transmission of the communications and is used during the entire time of this transmission.

The purpose of control apparatus is to act on the switching

apparatus in such a way that the latter transmits the communication
in a specified direction. It follows therefore that the control
apparatus can be engaged for a much shorter time than the switching
apparatus.

Consequently, in the second group of problems we can single
out three principal problems: the construction of the communication
network, the switching system, and the control system.

For broadcasting and television, all these problems are solved
relatively simply, since the transmission is from one source to many
consumers, located over a definite area, and no automatic switching
is necessary. For telephone and telegraph communication, this
problem becomes more complicated because of the need for insuring
transmission of messages between any two subscribers. The problem
is particularly complicated for telephone communication, where the
number of subscribers is so large, and a connection must be estab-
lished rapidly in simultaneous transmission of communications in
both directions.

A construction of reliable communication networks, particularly
for the USSR, is possible only when all the   connection pro-
cesses are automatized, and also when unattended amplifier points,
telephone stations, telegraph stations, and radio stations are built
with suitable remote control, remote signalization, monitoring,
and automatic regulation of the parameters of the channels and
the apparatus. Automatization of communication and the creation of

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unattended points and stations make it possible not only to increase the productivity of labor, but in many cases to reduce considerably the capital expenditure by reducing the volume of building, and in some scases by a more rational construction of the network.

Automatization of communication is primarily a question of producing automatic control systems for the switching apparatus and for other communication apparatus. It is therefore no wonder that the question of creating control systems has developed into a separate problem precisely because of the wide adoption of automatization.

The problem of creating an optimum control system is much similar in its formulation to problems in telemechanics, for here one solves many problems on the transmission of a number from the subscriber to a station, within the station, and also between stations, as well as the problem of creating control circuits for the station. It does, however, have its own peculiarities. The principal problems in control apparatus is the receipt from the subscriber of "an address" to direct the communication (the number of the called station), the determination of the unoccupied channels in the necessary direction, the transmission of the order to the switching circuit, the establishment of the connection, and if the connection is not complete, the further transmission of the number. On the other hand, the control apparatus must insure automatization of the verification and monitoring of the state and operation of both individual installations and lines (including

the control apparatus itself), as well as of the systems as a whole.

Consequently, control apparatus performs several specific functions, different from the functions of the other apparatus. At the same time, it is connected with the switching and other apparatus, as well as with the communication channels.

In the problem of constructing control systems one can separate out two basic groups of problems, that of designing control-apparatus circuits and the creation of systems for transmitting the control orders over the communication channels.

The present book is devoted to an examination of problems in the first group.

Modern control devices used in telephone, telegraph, and radio are made up of apparatus of discrete action. The elements most extensively used in this apparatus are electromagnetic relays and selectors. Recently all kinds of contactless relay-action elements have come into use -- semiconductor diodes and transistors, electron and ion tubes, ferrites, etc.

The process of creating any relay device consists of the following fundamental stages:

1) Structural synthesis, consisting of making up the structural diagram -- the principal diagram without indication of the parameters of the elements contained in it -- on the basis of the operating conditions of the network (on the functions it is to perform) and the choice of elements.

2) Calculation of the parameters of the elements that enter into the circuit, for which purpose a wiring or principal diagram is made up.

3) Wiring of the units and completion of the device.

The correctness with which these stages are performed is verified by an analysis of the operation of the circuit during individual stages of the construction.

The initial data for the design of the circuit of any device whatever are the conditions of interaction between this device and others, as well as the elements that can be used to build this device: relays, vacuum tubes, semiconductor diodes, transistors, ferrites, ionic devices, etc.

The device may have to satisfy other requirements, for example, those involving the parameters of the power source, dimensional requirements, various economic requirements, reliability requirements, immunity against noise, stability against mechanical action and against changes in temperature and humidity, and technological feasibility of manufacture. All these requirements may influence to some extent both the choice of the elements and the construction of the circuit.

Each stage in the synthesis has its own specific nature and calls for different steps. While the last state - wiring - is a purely manufacturing process, in the first two stages the correct choice of elements is determined by suitable computation.

From the list of problems discussed, the most extensively developed have been procedures for calculating the characteristics of individual elements, including electromagnetic relays, as given in the works by V.I. Kovalenkov, B. S.Sotskov, M. I. Vitenberg, V. A. Govorkov and others. Much experience has also been accumulated on the compilation of wiring diagrams and on wiring.

The synthesis of the structure of networks (the specification of the operating conditions of the network, the choice of elements, the determination of the necessary number, and the compilation of the most economical connection diagram), as well as analysis (the disclosure of the operating conditions of the completed network) began to be theoretically treated only during the last ten or fifteen years. Theoretical methods are not used even at present, and new networks are built in analogy with existing ones. In this case each step of transforming the network should be accompanied by a verification of its operation, and this complicates the problems of synthesis and analysis considerably. It must be added, that in such an "intuitive" construction sight is frequently lost of the criterion of necessity and sufficiency.

There are several methods for developing the structure, differing in their procedures and sequence of operations, as well as in the mathematical formalism used.

In the present book we shall consider some of the scientific methods for structural synthesis of radio networks, based on the

theory of relay-contact networks, and   we shall lean mostly on

methods that can be recommended for engineering practice, and are

aimed at creating the simplest possible networks.

In the general form, the structural synthesis of a network of

any device or center reduces to the following:

1) Determine the operating conditions of the receiving elements

(incoming external actions) and actuating circuits (transmitted by the

action circuit) from the specified conditions of the operation of

the network for definite elements.

2) Determine the number and character of memory elements (inter-

mediate relays), which must be introduced into the network to realize

the specified conditions.

3) Find the operating conditions of individual intermediate

elements (action circuits)   and the dependence of the actuating

circuits on the receiving and intermediate elements.

4) Make up several versions of the network structure and choose

the most applicable one by comparison.

In some cases one specifies not an element, but a set of

elements, and one determines during the synthesis process which of

these are best used for the construction of a network.

Finally, a case may occur when during the synthesis process

it is found advantageous to change the specifications or to change

the character of the receiving or transmitted action.

If the circuit must perform a large number of functions, it is

advantageous to break them up into individual groups, which can be performed by individual centers of the network. Such a subdivision of a complicated network into individual centers makes it possible to reduce the problem of creating a complicated network to that of designing several simpler networks. This may increase somewhat the number of elements in the network, but it simplifies its design. One establishes simultaneously methods of connection between the individual centers (their action on each other), and also methods of external action on the network produced and methods of having the network act on other devices and instruments. This serves as a basis for formulating the operating conditions of the individual centers of the network, after which the circuits made up of these centers are synthesized.

Many researchers and engineers have expended considerable work on the creation of scientific methods for structural synthesis of relay networks, mainly devoted to generalization and ordering of the "intuitive" methods of network construction. Thus, one can point to work by several Austrian and German electrical engineers, dating back to the beginning of the 20th century, (by R. Edler, M. Bode, and others). Somewhat later, in the 20th, the Soviet scientists M. G. Tsimbalistyy /3/ and A. K. Kutti /4/ have developed at the Leningrad Experimental Electrotechnical Laboratory, headed by V. I. Kovalenkov, methods that make it possible to construct a relay system for specified

conditions. However, the lack of mathematical means at that time has made such methods cumbersome and these have therefore not found wide application.

It became possible to lay the ground work for the theory of relay circuits only after one of the branches of mathematical logic -- algebraic logic /5, 6/ or Boolean algebra (and named after the British mathematician Boole) -- has been applied to contact networks. An important role was played in the development of mathematical logic by the Russian scientists, P.S. Poretskiy and I. I. Zhegalkin and later by A. N. Kolmogorov, A. A. Markov, P. S. Novikov, S. V. Yablonskiy, S. A. Yanovskaya /7, 8/ and others.

(The possibility of employing algebraic logic for relay circuits was first indicated by the Russian physicist P.S. Ehrenfest in 1910 in his review /9/ of the book by L. Kutyura "Algebraic Logic" /5/.)

The hypothesis of the possibility of creating an algebra for contact networks was confirmed in an unpublished paper "Algebra of Relay Circuits" written by the Soviet physicist V. I. Shestakov in January 1935,   which served as a basis for his dissertation /10/. V. I. Shestakov has shown that a contact network can simulate functions of algebraic logic, and the truth and falseness of the predictions are simulated by closed or open states of the circuit, while disjunction and conjunction of the judgements are simulated respectively by parallel and series connections of contact circuits. Analogous deductions were obtained at the same time (1936 -- 1938)

by the Japanese Nakashima /11/ and the American C.E.Shannon /12 — 15/.

On the basis of this mathematical formalism and the generalization of the experience in the synthesis and analysis of relay networks, M. A. Gavrilov laid the ground work for the theory of relay-contact networks and its practical application in several papers, published in 1945 — 1949, as well as in his monograph /16/, which was the first book in the world devoted to this problem.

Recently there has been a great expansion in work in the field of the theory of relay circuits, and its application. Following the book by M.A.Gavrilov, translated into Czech, German, Chinese, and Polish, several papers have been published concerning this theory /17 — 21/, including the book by V. N. Roginskiy and A. D. Kharkevich /18/ published in the USSR and translated into Chinese.   The number of works in the field of theory of contact and relay circuits is continuously increasing /22/, and they include a large number of Soviet papers (bibliographies for recent years are published periodically in the journal "Avtomatika i telemekhanika" /Automation and Remote Control/ /23/." We are unable to discuss all the works, but we shall start on a few of general character /24 — 75/.

The most highly developed part of the theory is the theory of contact parallel-series circuits, where there exists a possibility of mutual single-valued transition from a circuit to a formula, expressing not only the action of this circuit, but also its structure.

By suitable algebraic transformations of formulas, one can obtain different versions of a network, interconnect contacts, redistributed contacts over the relays, and simplify the circuits, on the basis of the condition that individual states are not encountered during the process of operation of the network. One can find here, for individual parallel-series networks, the simplest solutions as regards the number of contacts.

In the general form, the problem of finding the most economical network has not yet been solved theoretically and is of great interest and tremendous difficulty.

Less developed are bridge circuits, which, experience has shown, are simpler in many cases for the same conditions.

In many works, including those of M. A. Gavrilov /16, 69/, and A. M. Brileyev /70/, methods are given for separating bridge elements after constructing parallel-series contact networks. M. A. Gavrilov /69/ has formulated the conditions for the possibility of separating a bridge element. A considerable step forward in the transformation of bridge circuits was the application, by A.G. Lunts /30 -- 32/, B. I. Aranovich /71, 72/ and M. L. Tsetlin /61, 74/ of matrices for recording the structure and action of contact networks. A matrix calculus for contact networks was also used by Hohn and Schissler /75/, Piesch /40/, Seshu /76/, and a few others.

In bridge contact networks, the number of contacts can be reduced by introducing valves, while the methods of M. A. Gavrilov

and A. G. Lunts, referred to above, make it possible to determine the points in which the valves should be connected. M. A. Gavrilov /77/ has developed the theory of construction of so-called rectifier grids, which made it possible to reduce the number of contacts in each relay to one double-throw contact. Another analytic method of the construction of bridge-type (1, k)-pole network was proposed in 1955 by G. N. Povarov /41, 44/. This method, called the "cascade method" uses for its construction the separating properties of contact pyramids (trees).

Finally, notice should be taken that in recent years methods have appeared of constructing contact networks, which do not use the formalism of algebraic logic directly, although the theories of contact networks have been used in their development. One such method is that of the indeterminate functions, developed in Czechoslovakia by F.Svoboda /59, 78/. The method consists of selecting successively elementary circuits, which satisfy the conditions of conductivity from the outputs to the input and to other centers. This includes also the graphical method developed by G. N. Povarov /43/ for symmetrical contact (1, k)-pole networks, and a graphic method developed by the author /45 -- 47/ and detailed in the present book.

Along with studying the general properties of contact networks, of practical interest is also the study of the so-called ordered-type networks. These networks include above all symmetrical and functionally-separable networks, which have been considered by

many authors, including Shannon /12 — 15/, M. A.Gavrilov /16/, and
G. N. Povarov /41, 42, 43, 79/. Of great importance for communication
devices are switching networks, which, as shown by A. D. Kharkevich
/80/, represent an independent group of ordered contact networks.

One of the shortcomings of modern theory of relay-contact net-
works is the fact that it is applicable, in accordance with the
terminology of Gr. C. Moisil /81, 82/ to ideal contacts, which change
their states instantaneously. In practice, during the time of opera-
tion or of release of any relay, its contacts close and open not
simultaneously. There are many papers devoted to a study of this
problem, but there are still not good enough engineering methods for
taking into account the non-simultaneity of operation of contacts in
the synthesis of networks.

Along with a study of networks with contact-making two-position
relays, works have also appeared devoted to networks with multi-
position element contacts (such as selectors). V. I. Shestakov
/83/ showed as early as in 1946 that   one can apply to such
networks the formalism of n-valued calculus of propositions. One
can indicate also later works by V. M. Ostianu /84/ and V. I.
Shestakov /85/, pertaining to such networks. In the present paper
            graphic
we give a method of constructing networks made of contacts of
multi-position elements.

Everything mentioned above pertains only to contact networks.
The problem of synthesis of relay networks, i.e., the

networks containing the windings of the relays, and also resistances, have been very little developed. Individual mentions of transformation of relay networks are found in the papers by V. I. Shestakov /86 — 88/, M. A. Gavrilov /16/ and many others /18, 63, 89, 90/, but all these pertain essentially to so-called normal networks, i.e., networks in which the contact circuits are connected to one pole of the battery while the relay windings are connected to the other.

Transformations connected with introducing of bucking windings into relays, have been developed by A. N. Yurasov /64, 65// and D. I. Shnarevich /91/. The author has considered the possibility of transforming such networks, in class π /51 — 53/, for certain cases of ratios of the relay parameters. T. L. Taystrova (still unpublished) has shown that in this case one can employ for the transformation of relay networks the mathematical formalism of logic of classes (multiple-valued logic). Further development of the theory is necessary in this direction, so as to create a general theory of transformations (and then of construction) of relay networks with parametric relationships, i.e., to produce a mathematical formalism for the transformation of relay networks.

It is also necessary to study the use of such elements in relay networks, as capacitors, which have the ability to store energy /50/.

The analytical method for writing down the structure has also

made necessary the development of corresponding methods of writing down the conditions. The best method of recording operating conditions of multi-contact relay networks are the so-called connection tables, widely used by M. A. Gavrilov and others. The author has formulated conditions of realizability of connection tables /18, 49/, while V. G. Lazarev /92 — 94/ obtained estimates of the minimum number of intermediate relays, that must be introduced in order for the table to become realizable.

A somewhat different method of tabular recording of the operating conditions of a network has been proposed by Huffman /95, 96/. M. L. Tsetlin proposed the use of matrices of states and reactions /62/ for writing down the operating conditions of multi-step networks. Many authors, for example, V. I. Shestakov /85, 97 — 99/, and Gr. C. Moisil /36/ and others employ algebraic methods for writing down the conditions. Recently V. I. Shestakov proposed for this purpose the use of special punched cards /100/, on which unique connection tables are made up.

One of the features of the operating conditions for relay networks is that a certain circuit may be indiffirently closed or open in certain states of the network. In particular, this pertains to so-called unused states /48/, i.e., states which are not encountered during the process of operation of the network. There exists several methods of taking these states into account. In particular, M. A. Gavrilov has introduced special equivalences

for specified sequences of relay actions in the network. In the present book we give a system, developed by the author /18, 49/, for recording such states in the connection tables and in the formulas, as well as methods of changing over from conditions to networks with allowance for indifferent states. Certain operations of three-valued logic are used here.

It should be noted that the existing methods of writing the conditions cannot always be employed as yet. In particular, certain networks must realize such a number of sequences, that they cannot be written down in practice in the form of connection tables. Thus, for example, if a network has ten inputs and ten outputs and actions enter through the inputs in any sequence, while the network must give out the actions in the outputs in the same sequence as they are received in the inputs, except with a certain delay, it is almost impossible to write down these conditions, since it is necessary to construct $10!$ different connection tables (this example was borrowed from M. A. Gavrilov).

It is also of practical interest to investigate other networks with ordered sequences of action (in analogy with ordered contact networks). Such include, for example, cyclic /101, 102/, and counting /103, 104/ networks, which have extensive application in control networks. In the general case, however, control networks are not networks of the ordered type, and not much attention will therefore be paid to them in the present book.

In analogy with single-step networks, one can speak in the case of multiple-step /sequential/ networks, apparently, of unused sequences of relay operation or of external actions. There exists as yet no general method of taking such sequences into account in the synthesis of networks, and everything reduces essentially to the disclosure of unused states.

The task in the synthesis of a relay network consists of not merely obtaining a network corresponding to specified working conditions, but satisfying in addition several other requirements. Let us dwell on some of these requirements.

1. The production of networks with a minimum number of elements, particularly if it is considered that the number of contact springs on each relay is limited. The minimum criteria for contact networks have not yet been established, and therefore it is necessary in the synthesis of networks to resort to a comparison of versions. However, certain recommendations on the simplification of networks do exist and are given in the present book.

2. The production of reliably-operating networks. This problem has not been developed at all, and there are even no criteria by which to judge the reliability of a given network. Note should be taken in this respect of the paper by Shannon and Moore /105/ on the construction of reliable networks from less reliable elements.

3. The construction of technological networks, i.e., networks with only one type of relay and standard wiring.

Satisfying this requirement to some extent are the cyclic networks considered by V. I. Ivanov /101, 102/.

4. Use of contactless elements in relay networks.

A feature of relay networks is that one and the same requirement and one and the same set of conditions can be realized by a large number of networks. Therefore the choice of versions becomes very difficult. This leads to the tendency of mechanizing the process of constructing and analyzing networks. Attempts at using universal computers for this purpose were not crowned with success, and therefore the scientific thought has been devoted to the creation of specialized machines.

Mention should be made here of logical machines developed in 1951 in England /106/ and in 1954 in Austria /26, 27, 108/, which made it possible to analyze single-step contact networks. The machine of Shannon and Moore /107/, developed by the Bell System (U.S.) in 1953 , makes it possible not only to analyze the network, but to discover excessive elements in the network. In 1954, T.T.Tsukanov /109/ produced an apparatus which makes it possible to determine, for a complicated contact network, all paths between selected terminals.

In 1955 — 1957, engineer P. P. Parkhomenko /24, 110/ developed at the Institute of Automation and Telemechanics, Academy of Sciences, USSR, a machine for the analysis of the operation of multiple-step /sequential/ relay networks. (This machine was demonstrated at

the World's Fair of Brussels in 1958 and was awarded the highest prize, together with a machine for synthesis).

The question of automatization of the synthesis process for contact and relay networks has remained unsolved for a long time. The main obstacle to the creation of such a machine was the absence of a sufficiently good regular method for network synthesis.

In 1955-1956, engineer F. Svoboda of the Institute of Mathematical Machinery of the Czechoslovak Academy of Sciences, constructed, on the basis of a combinatorial method for the construction of contact networks /57 — 59/ developed by him, a semi-automatic machine for the synthesis of single-step contact multipole networks.

At the end of 1955 the author, in the Laboratory on the Development of Scientific Problems for Wire Communication, Academy of Sciences, USSR, proposed on the basis of a graphical method a machine /2, 111/ for the synthesis of networks of contact (1, k)-pole networks. (A sample of the machine, developed at the Laboratory for Wire Communication, Academy of Sciences, USSR and produced by the machine shop of theInstitute of Automation and Telemechanics, Academy of Sciences, USSR, was exhibited at the World's Fair in Brussels in 1958 and was awarded the highest prize together with the machine for network analysis.) Later on the possibilities of operation of this machine were expanded by adding devices for introduction of tasks in the form of switching tables and for auto-

matic choice of the necessary number of intermediate relays and
their operating sequence /112/. This is the first of known machines
for complete automatization of the process of construction of relay
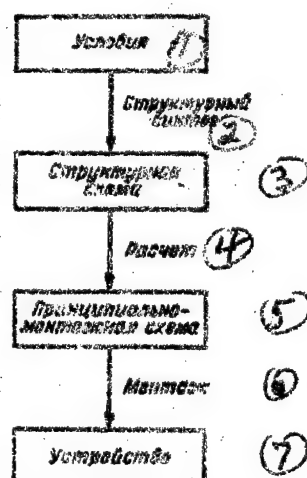networks.

In conclusion,   it must be noted that the theory of relay-
contact networks is finding increasing application also in relay-
action networks with contactless elements.

In a book published under the editorship of G. Aiken /113/,
a special symbolism is developed which can be called a unique algebra
of electronic switching networks. Analogous methods have been used
also by many authors for networks with contactless elements /34, 39,
44, 62, 114, 115, and others/. However, there are no general prac-
tical methods for the creation of relay networks with contactless
elements.

The development and generalization of the research being
carried on at the present time should lead to the creation of a
general theory of construction of discrete-action networks. In
the present paper we touch upon only individual problems, connected
with the creation of relay control networks, as applied to telephone
devices.

The available experience of application of theoretical methods
for the analysis of existing   control circuits for manual
and automatic telephony and for synthesis of new networks shows
that these methods make it possible to obtain networks that have

10 to 30% fewer elements /18, 134/.

КEY:

1) Conditions;
2) Structural synthesis
3) Structural diagram;
4) Calculation;
5) Principal wiring diagram;
6) Wiring;
7) Equipment.

Chapter 1

BASIC DEFINITIONS

1. RELAY NETWORKS AND THEIR PARTS

By relay network is meant a device containing a certain number
of electromagnetic relays and two or more position elements (switches,
pushbuttons, double-throw switches, etc.), and performing certain
functions. Each relay network (Fig. 1) is connected with some other
devices (including other relay networks) and its task is to transform,
in accordance with a given program (in accordance with specified
conditions) certain signals received by this network and transmitted
in modified form.

The signals can be introduced into the relay network both
through electric circuits, in the form of changes in certain elec-
tric parameters, to which the receiving relay responds, or with
the aid of keys, pushbuttons, and other elements, which respond
to mechanical, thermal, auditory, and other external factors. These
circuits and elements will henceforth be called receiving. The
main feature of receiving elements is that their responding or-
gans (windings of relays, knobs of keys, etc.) receive external
signals (relative to the given network), and all that enter into the

network directly are their actuating organs (contacts).

A relay network produces output signals as a rule via electric actuating circuits, but the circuit may contain also other actuating elements such as electromagnets, tubes, signalling devices, etc. The feature of actuating elements is that only their reacting organs enter into the network.

To transform the signals into establishing a connection between the receiving elements and the actuating circuits, the network may contain intermediate relays, the action of which is determined by the states of either the receiving elements or the intermediate relays themselves. In some cases the relay network may include resistances, rectifiers, capacitors, and other electrical circuit elements.

It should be noted that the same relay may serve both as a receiving and an intermediate relay. The same can be said of circuits: one and the same circuit may during different periods of the network operation be either receiving or actuating.

A relay network may be broken up into several smaller sub-networks or into individual circuits. In each relay network one can separate out a contact network, i.e., a multipole network, consisting of only contacts of receiving elements and intermediate relays.

Depending on the relative connections between the contact networks and the relay windings, relay networks are divided into three principal groups: normal ones, in which the relay windings in the actuating elements are connected to the same pole of the

battery, while the contact circuit is connected at the other (Fig. 2a); inverse, in which the windings of all relays and actuating elements are connected in series between the   terminals of the battery, while the contact circuits are connected to the windings (Fig. 2c), and mixed.

The points of connection of the individual elements or network circuits will be called nodes, while nodes to which external wires (relative to the given network) are connected will be called poles.

In the general case, a relay network represents a multi-pole network. However, in most cases the entire network or that part of it connected to the intermediate relay can be considered as a two-terminal network, in which the poles are the points where the current source is connected.

2. States of a Relay Network

Each network made up of n 2-position elements (relays, keys, etc.) can have

$$L = 2^n \qquad (1.1)$$

different states. To number these states of the network we employ the following method. To each element in any sequence we assign a number i (i = 1, 2, 3, ..., n) and a "weight" $q_i = 2^{i-1}$, i.e., 1, 2, 4, ..., 2^{n-1}. The number of the state is assumed to be the sum of the "weights" of those relays, which operate in the

given state.

Thus, all the 2^n states will have numbers from zero (not one relay operates) to 2^{n-1} (all relays operate). Table 1 gives an example of all $2^3 = 8$ states of a network with three elements A, B, and C and the numbering of the states when these elements are numbered in their alphabetical order.

Table 1

	A	B	C	
i	1	2	3	
q_i	1	2	4	
	−	−	−	0
	+	−	−	1
	−	+	−	2
	+	+	−	3
	−	−	+	4
	+	−	+	5
	−	+	+	6
	+	+	+	7

1) Elements, 2) numbering of states, 3) states of elements.

3. PARALLEL-SERIES AND BRIDGE NETWORKS

Depending on the mutual interconnection of individual elements of a relay network, we distinguish between two classes of networks.

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The parallel-series class of networks (class Π) includes all networks whose elements are connected to each other either in series or in parallel. For example, the networks in Fig. 3 belong to class Π.

Networks of the class of non-parallel-series networks (class H), contain in addition to series in parallel connections, also bridge connections. In the presence of bridge connections in the network, we cannot concerning certain elements (the bridge elements) that they are connected in series or in parallel relative to other elements.

When direct current is connected to the poles of the network, the current flowing through each of the elements of a network of class Π can be in only one direction. In class H networks, on the other hand, there exists for the case of bridge elements the possibility of current flowing in two opposite directions, depending on the state of other elements in the network.

An example of a network of class H is shown in Fig. 4a, which contains the bridge element e.

Bridge networks, in turn, are divided into planar, i.e., those which can be drawn on a plane without crossing the wires, and non-planar, which cannot be drawn without intersections. An example of a non-planar network is shown in Fig. 4b.

4. CONTACT NETWORKS

Like relay networks, contact networks represent multi-pole networks and in most practical cases are oriented, i.e., one can separate in them the "inputs" from the "outputs," and the network itself can be represented in the form of a (p, k)-pole network, having p inputs and k outputs (Fig. 5).

In such a multi-pole network, we are interested essentially in the circuits between the inputs and the outputs. A particular case of such an oriented multi-pole network is a $(1, k)$-pole network with one input and k outputs, or a $(k, 1)$-pole network with k inputs and one output.

Although in general it is immaterial which of the poles is an input and which is an output, we shall agree, in the case of dc networks (where necessary to make the distinction), to consider the inputs the terminals of the network connected to the plus of the battery.

If it is impossible to produce circuits in a (p, k)-pole network between any of its inputs (or outputs), such a network is called isolating on the input (output) side.

An example of an isolating $(1, k)$-pole network is the contact pyramid (Fig. 6).

The number of different contact 2-pole networks which can be made up of the contacts of n relays is

$$(1.2)$$

i.e., when n = 2 there are 16 circuits (2-terminal networks), when n = 3 we have 256, when n = 4 we have 65,536, etc.

All these circuits can be joined into groups, in which one network is obtained from the other either by changing the names of the relays, or by changing the closing contacts of one relay by opening contacts of another, and vice versa. It was demonstrated by Polya /116/ that there will be 22 such groups when n = 3, 402 groups when n = 4, etc.

Networks of one group, constructed in the same manner, will be of the same complexity, i.e., have the same number of contacts on each relay. For example, the networks of Figs. 7a and b belong to one group, since the network of 7b is obtained from the network of 7a by renaming relays B and D and by replacing the contacts of relays A and C by their opposites.

Thus, if all the networks cannot be covered in practice when n 3, by breaking them up into groups one can investigate all of them separately for n = 3 and n = 4, as was done by G. N. Povarov /41/, and by Higgonet and Grea /19/.

Of all the networks that can be produced of contacts of n relays, we shall separate the one consisting of a series connec-tion of one contact each (normally closed or normally opened) of all relays. There will be 2^n such circuits and each of these will be closed only in one definite state of the network. In other words, to each of the 2^n states of the system we can set in

correspondence a circuit which is closed only in that state.

The algebraic expression corresponding to this circuit is called the constituent (see Chapter 4, Section 3).

5. ORDERED CONTACT NETWORKS

Of practical interest is the study of individual types of ordered networks, which are characterized by some feature or another. In addition to the pyramids indicated above, there exists a large number of different ordered contact networks.

Let us note some of them.

1. Symmetrical networks, i.e., those in which the circuit is closed when a definite number of any s relays from a total number of n operate. The numbers s are called the working numbers of the relay. The symmetrical networks have been studied, in particular by C.Shannon /12 — /15, M. A. Gavrilov /16/, and G. N. Povarov /43, 44/.

In connection with the development of the graphic method of construction of networks, the question hasbeen raised of investigating the properties of partially-symmetrical networks (see Chapter 7).

2. Switching networks, i.e., (p, k)-pole networks that are isolating on both sides and have that property, that each input can be connected with any output (fully accessible networks) or with a specified part of the outputs (not fully accessible networks).

Devoted to a study of these networks, are , in particular, the papers by A. D. Kharkevich /80/ in connection with creating switching systems for automatic telephone stations.

A special type of switching network is a (k, k)-pole network, in which each of the inputs must always be connected with one and only one of the outputs, and the changes in switchings should satisfy specified conditions. Such networks are used in certain computing machines and in particular in the machine for the synthesis of contact (1, k)-pole networks.

3. A universal network, from which one can obtain any of the possible 2^{2n} networks by eliminating (shorting or disconnecting from the circuit) individual contacts. We shall meet with the construction of such a network when producing a tableau for the machine for the synthesis of (1, k)-pole networks (see Chapter 11).

6. ADMITTANCE OF CIRCUITS IN RELAY NETWORKS

One of the most important physical parameters of electric networks is their impedance or the reciprocal, the admittance. This parameter is particularly important in relay circuits, in which the operation of the relay depends essentially on the voltage and impedance of the circuits connected to the relay. We shall agree henceforth to speak of admittances rather than impedances, since it will be shown that it is more convenient for algebraic notation.

The admittance of any electric circuit depends on the admittances of the elements contained in the circuit and on their interconnection. The admittances of individual elements are determined by their physical properties and can be either constant or dependent, for example, on the frequency, on the applied voltage, on the temperature, or other extraneous factors.

In relay networks we usually deal with direct current, and therefore we shall consider the properties of the fundamental elements (contacts, rectifiers, and windings) used in these networks under direct current.

As is well known, the overall admittance Y of a circuit consisting of elements A and B with admittances Y_A and Y_B in parallel is the sum of these admittances

$$Y_{par} = Y_A + Y_B. \tag{1.3a}$$

and   in the case of a series connecting they are determined from the following formula

$$Y_{ser} = (Y_A^{-1} + Y_B^{-1})^{-1} = \frac{Y_A Y_B}{Y_A + Y_B}. \tag{1.3b}$$

To simplify the notation we shall write instead of the admittance Y_A of the element A merely the symbol of this element. In this case formulas (1.3) can be rewritten

$$Y_{par} = A + B. \tag{1.4a}$$

$$Y_{ser} = (A^{-1} + B^{-1})^{-1} = \frac{AB}{A + B}. \tag{1.4b}$$

The expression A + B in Eq. (1.4a) can be considered not merely as an expression for the calculation of the overall admittance of the parallel admittances A and B, but also as a notation for the parallel connection of the elements A and B. Thus, a parallel connection is conveniently denoted with the plus symbol.

For a series connection of admittances, we do not have in general such a simple relation. We shall agree to denote a *series* connection of elements by multiplication sign, i.e., we shall write formula (1.4b) in the form

$$Y_{(A > B)} = \frac{AB}{A + B}$$

or, even simpler,

$$A < B = \frac{AB}{A + B}.$$

$$(1.4c).$$

Thus, the expression A x B indicates, on the one hand, that the elements A and B are connected in series, and on the other hand that their combined admittance is calculated from formula (1.3b).

In a relay network, the admittance between any two terminals is determined by the elements that are included in the circuit between these two terminals for each combination of the states of the contacts. Since a network with n relays can have 2^n such combinations, the admittance of a relay circuit can have up to 2^n different values, and the change in this quantity can occur only in jumps, during the instants when the states of the

individual relays change.

Let us consider now the changes in the admittances of relay circuits with different elements.

1. Admittance of Contact Circuits

The main element of a relay circuit is the relay contact, which can be either closed or opened. The admittance of a contact in the closed state will be considered to be infinite, and in the open state to be zero, i.e.,

$$Y_a = \begin{vmatrix} 0 - \text{open} \\ \infty - \text{closed} \end{vmatrix}$$

(1.5)

In the case of parallel or series connection of the contacts having the extreme values of admittance, the overall admittance can also assume only these two values, as shown in Table 2.

Table 2

Y_a	Y_b	Параллельное соединение $Y_a + Y_b$	Последовательное соединение $Y_a + Y_b$
0	0	0	0
0	∞	∞	0
∞	0	∞	0
∞	∞	∞	∞

1) Parallel connection, $Y_a + B$, 2) series connection $Y_a \times Y_b$.

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To simplify the notation and for convenience in changing over to algebraic transformations, we shall agree, following V. I. Shestakov /10, 86 — 88/ to denote an infinite admittance not by the infinity sign, but by unity (1). On the other hand, the admittance Y_a of the contact a of relay A will be denoted simply by a, i.e., we write expression (1.5) in the form

$$a = \begin{cases} 0, \\ 1. \end{cases} \qquad (1.6)$$

In order not to confuse this reduced admittance with the ordinary concept of an admittance of one mho, we shall call this a structural admittance.

In accordance with Table 2 and with the values from expression (1.6), we can find the following relations:

a) For a parallel connection

$$0 + 0 = 0; \ 0 + 1 = 1 + 0 = 1; \ 1 + 1 = 1;$$

b) For a series connection

$$0 \times 0 = 0; \ 0 \times 1 = 1 \times 0 = 0; \ 1 \times 1 = 1.$$

These relations correspond fully to the relations of Boolean algebra (for more details see Chapter 2). Since the series connection coincides in this case with Boolean multiplication, these symbols for series connection will be denoted with the usual multiplication sign (.)

The structural admittance of a contact network, consisting of contacts of n relays, depends on the state of the contacts

contained in this circuit. And since the contacts of n relays can be in 2^n different combinations (each relay either operates or does not operate), and the contact circuit for each of these combinations can be either closed or opened, consequently, the structural admittance of a contact circuit depends, on the one hand, on the connection between the contacts themselves (i.e., on the structure) and on the other hand on the states of the relays included in the network. Thus, for example, for the contact network shown in Fig. 3a, the dependence of the admittance on the states of the relays is shown in Table 3.

Table 3

	1) Реле		2) Проводимость цепи	3) Структурная проводимость
A	B/b	C/b		
Не раб.	Не раб.	Не раб.	0	0
Раб.	» »	» »	∞	1
Не раб.	Раб.	» »	∞	1
Раб.	»	» »	∞	1
Не раб.	Не раб.	Раб.	∞	1
Раб.	» »	»	0	0
Не раб.	Раб.	»	∞	1
Раб.	»	»	∞	1

1) Relay, 2) admittance of the circuit 3) structural admittance, 4) not working, 5) working.

Inasmuch as the admittance of the contact is independent of the direction of the current, the admittance of the contact circuit is independent of which pole is considered the input and which the output.

2. Admittance of Circuits with Rectifiers

In relay networks one makes extensive use of rectifier elements — semiconductor (cuprox, germanium, selinium, etc.) diodes, having a large admittance Y' (low resistance, on the order of several times 10 ohms) for one direction and low admittance Y'' (large resistance, on the order of tens of thousands ohms and more) for currents in the opposite direction.

currents in

Although the admittances of rectifier elements are finite, nevertheless, considering their action in relay networks, we can assume that the structural admittance of a rectifier element, depending on the direction of the current, assumes the values $Y' = 1$ and $Y'' = 0$.

In other words, the admittance of a rectifier element is oriented, i.e., it depends on the current in it.

For the rectifier K, conducting from pole 1 to pole 2 (Fig. 8), we obtain:

$$Y_{12} = Y',$$
$$Y_{21} = Y'',$$

or, going to structural admittances

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$$Y_{12} = 1,$$
$$Y_{21} = 0$$

(1.7)

A circuit which contains rectifier elements can also have an oriented admittance. Thus, for the circuit of Fig. 9a, the admittance will have the values shown in Table 4.

Table 4

A	B (Б)	C (В)	проводимость		структура приводимости		$Y_{12} = Y_{33}$	Y_{11}	Y_{33}
			Y_{22}	Y_{33}	Y_{12}	Y_{23}			
−	−	−	0	0	0	0	0	0	0
+	−	−	∞	∞	1	1	∞	∞	∞
−	+	−	Y_1'	Y_1'	0	1	$G_1 \times G_2$	0	$G_1 \times G_2$
+	+	−	∞	∞	1	1	∞	∞	∞
−	−	+	Y_2'	Y_2'	1	0	G_2	G_2	0
+	−	+	0	0	0	0	G_1	G_1	0
−	+	+	Y_2'	Y_2'	1	0	G_1	G_1	0
+	+	+	$Y_1' \times Y_2'$	$Y_1' \times Y_2'$	1	0	$G_1 + G_2$	$G_1 + G_2$	0

1) Relay, 2) diagram Fig. 9a, 3) admittance, 4) structural admittance.

* To simplify the notation we denote non-working relays with minus and working ones with plus.

3. Contact Circuits with Active Resistances

Relay windings and other active resistances have non-oriented finite admittance G, i.e., one satisfying the inequality

$$0 < G < \infty \tag{1.8}$$

The admittance of a circuit which contains contacts and elements of finite admittance can have both the extreme values zero and infinity, as well as finite values. Thus, for example, Table 4 lists values of the admittance for the circuit of Fig. 9b. The presence of rectifier elements in such a circuit makes the admittance oriented in this case, as shown in this table (the admittance of the rectifiers is taken to be correspondingly zero or infinity) for the network of Fig. 9c. The properties of the active resistances in relay networks will be considered in greater detail in Chapter 8.

7. SINGLE-STEP AND MULTIPLE-STEP NETWORKS

Depending on the character of their action, relay networks can be either single-step or multiple-step /sequential/.

Single-step networks are those in which the state of the actuating circuits is determined by the state of the signals at each given instant. In such circuits there is no provision for a sequential action on the part of individual elements in time, with the exception of those sequences which can arise in the network during the instants of operation or release of the relays

either due to a spread in their time parameters (in simultaneous operation of several relays), or due to non-simultaneous closing and opening of the individual contacts.

If we denote by x_1, x_2, ..., x_k the states of the receiving elements, and by z_1, z_2, ..., z_p the states of the actuating circuits, then in a single-step network the states of any actuating circuit z_i are a function of the states of the receiving elements only, i.e.,

$$z_i = f_i(x_1, x_2, \ldots, x_k). \tag{1.9}$$

In multiple-step networks, the states of the actuating circuits depend not only on the states of the external signals at a given instant, but also on the sequences of these signals, i.e.,

$$z_{i,t} = f_i(x_1, x_2, \ldots, x_k, p_{t-1}), \tag{1.10}$$

where p_{t-1} is the state of the network (intermediate elements) during the instant preceding the given signal.

Since in a multiple-step network the sequence or sequences of signals is of prime importance, such a network must contain memory devices in the form, for example of intermediate relays.

During the process of operation of a multiple-step network, its elements change their states both as a result of external signals, and as a result of internal interactions. The sequential combinations of the states of the elements of the network, differing in the state of at least a single element, will be called steps.

The period of operation of a network, during which all the elements return to the state assumed to be the initial state, will be called the cycle of the operation of the network. Depending on the conditions of the external signals, networks can be designed for a single definite cycle, but in practice one encounters as a rule networks with several different operating cycles, depending on the sequence of receipt of external signals.

Since the change in the state of the elements of the network depends both on the change in the external signals and on the internal interactions, we shall distinguish two types of steps during the process of operation of a multiple-step network /18/:

1) Steps in which the change in the state of the elements upon going to the next step is due to changes in the external signals. It is clear that the duration of such a step is independent of the parameters of the network itself or of its elements. Such steps will be called stable.

2) Steps in which local circuits are produced, due to the operation or drop-out of a certain relay from this network. The duration of such a step is determined only by the operating time or the release time of the corresponding relay. Such steps will be called unstable.

The operating period of a network between two changes in external signals will be called a stage in the operation of the network. A stage may consist of one or several steps.

The control networks used in communication devices are as a rule multiple-step networks with several operating cycles. Single-step networks are encountered only as individual units in multiple-step networks. Therefore principal attention will be paid to multiple-step networks.

Along with a general study of multiple-step networks, practical interest attaches also to multiple-step networks of ordered type, in which the sequence of action of the relays obeys a certain prescribed law. Such multiple-step networks include above all networks with a single receiving element. In these networks, particular interest are cyclic networks /102, 103/, which includes counters, distributors, and other devices.

Another important group of ordered networks are the so-called autonomous networks, which operate without external signals. These include, in particular, pulse networks (pulse generators). A separate group is made up of clamping networks.
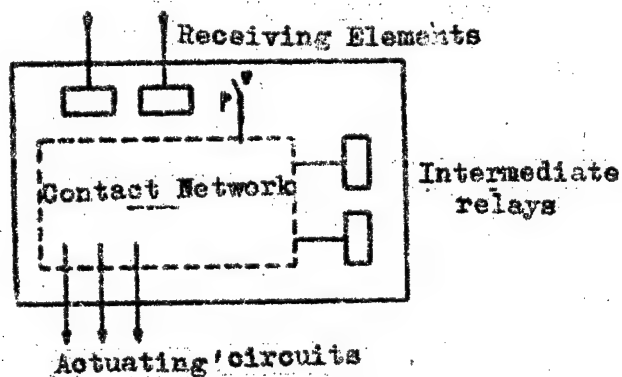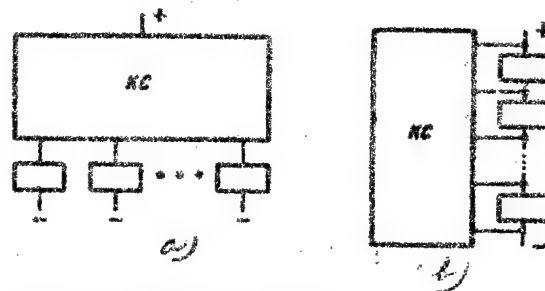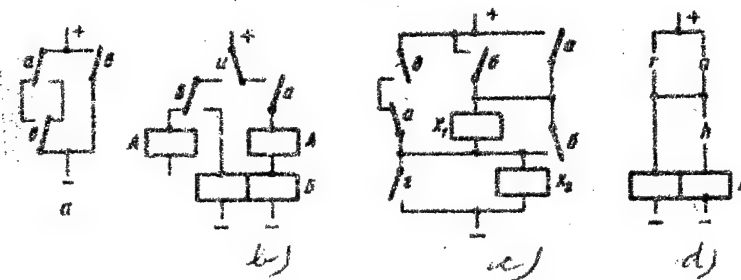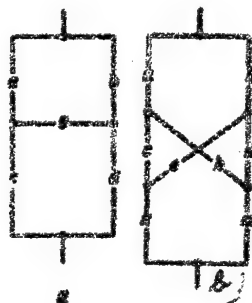

Fig. 1.

KC = Contact network
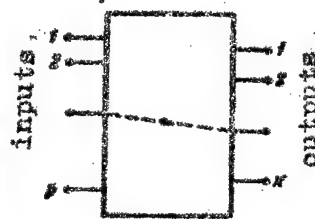
Fig. 2

Fig. 3.
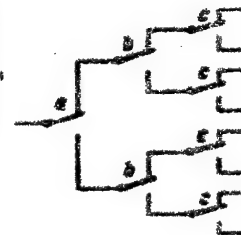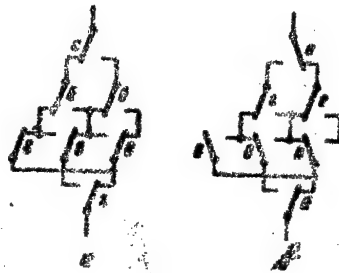
Fig. 4.   Fig. 5   Fig. 6
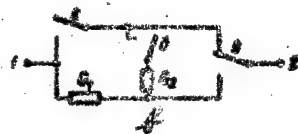
Fig. 7.

Fig. 8.

Fig. 9.

Chapter 2

NOTATION FOR THE STRUCTURE AND OPERATING CONDITIONS OF A RELAY NETWORK

1. NOTATION FOR THE STRUCTURE OF RELAY NETWORKS

As already indicated, by structure of a relay network is meant its composition (the elements contained in it), the mutual placement of the elements, and the connections between them.

One of the most widely used methods of notation for the structure of the network is graphic notation in the form of principal or wiring diagrams, in which separate elements of the network are represented by arbitrary symbols, and the characteristics of these elements are denoted with numbers, words, or some other arbitrary symbols. There exist many different ways of graphic representation of networks, but we shall not deal with them here.

In some cases it is convenient to designate the elements of a relay network with letters. It is most customary to denote relay windings with capital letters (A, B , X_1, X_2) and the contacts with corresponding lower-case letters (a, b , x_1, x_2, ...) and opening (normally-closed) contacts are distinguished from

/break/

/make/.

closing (normally-open) contacts with a superior bar (i.e., $\bar{a}$, $\bar{b}$, $\bar{x}_1$, $\bar{x}_2$, ...). If the relay has several windings which must be distinguished from each other, we number (the order of numbering is immaterial) these windings and agree to write the number in the form of a superscript (i.e., A^1, A^2, ...). In cases when the individual windings are so connected that their action may cancel each other (opposing windings), we shall agree to distinguish the windings, in which the current flows from the end of the winding to the start of the winding, by an arrow above the letter (for example A and $\overleftarrow{A}$, or A^1 and $\overleftarrow{A^2}$). We can introduce symbols also for other elements.

If we now denote by the addition sign ($+$) a parallel connection of network elements and by the multiplication sign a series connection, then the structure of any network of class π can be uniquely recorded in the form of an algebraic expression or a structural formula.

Thus, the networks of Fig. 10 can be represented by the following structural formulas:

a) $\quad F = u\,(\bar{b}A + бБ) + \bar{u}aAБ + бX;$

b) $\quad F = (u + aБ)[(\bar{\bar{u}} + \bar{b})\,A + бБ] + бX;$

c) $\quad F = [\bar{b}A + бБ + \bar{u}\,(AБ + \bar{a})]\,R + бX;$

d) $\quad F = (u + aAБ)(A + б\overleftarrow{A}Б) + бX.$

If is, however, impossible to write down a network of class H by means of a structural formula, for in these networks there exist elements of which one cannot say whether they are

connected in parallel or in series, i.e., one cannot use the sign of addition or multiplication.

The most convenient form for writing down the structure of networks of class H   are the so-called structural matrices (or matrices of direct admittances, as they are sometimes called).

To   make up the structural matrix we number the nodes of the network in any sequence. It is not essential here to number the nodes that enter into some circuits of class $\overline{\Pi}$, but all the poles must be numbered. One then constructs a quadratic matrix with number of rows and columns equal to the number of numbered nodes of the network, and at the intersection of the i-th row and the j-th column one writes down the symbol for the circuit from the node with number i to the node with number j, not passing through any other numbered nodes. Then the circuit will be called the direct circuit ϕ_{ij} between the nodes i and j.

Thus, the structural matrix with q nodes will have the form

$$
\begin{bmatrix}
\varphi_{11} & \varphi_{12} & \cdots & \varphi_{1q} \\
\varphi_{21} & \varphi_{22} & \cdots & \varphi_{2q} \\
\cdot & \cdot & \cdots & \cdot \\
\cdot & \cdot & \cdots & \cdot \\
\varphi_{q1} & \varphi_{q2} & \cdots & \varphi_{qq}
\end{bmatrix}
$$

If there is no circuit passing through other nodes between the nodes i and j, one writes in the corresponding cell $\phi_{ij} = \phi_{ji} = 0$ (null), corresponding to an open circuit (zero admittance).

If, however, two nodes i and j are connected directly, then we assume $\phi_{ij} = \phi_{ji} = 1$ (infinite admittance). It is   obvious that for any node $\phi_{ii} = 1$.

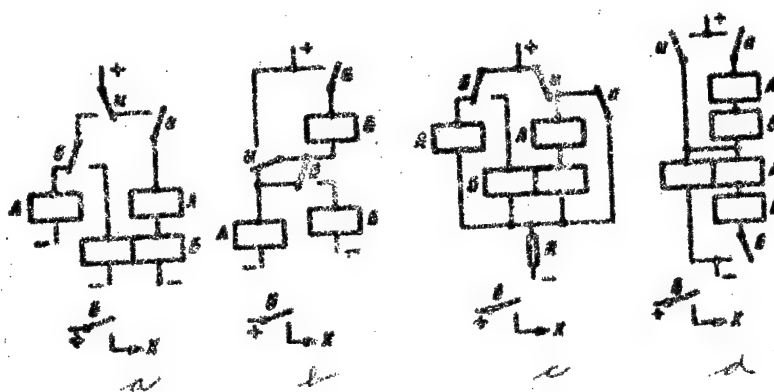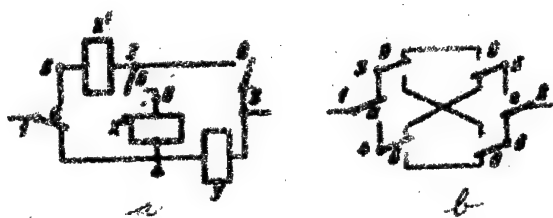Thus, the structural matrix of a network will be square, with

Fig. 10.

Fig. 11.

$$\begin{bmatrix} 1 & \varphi_{12} & \cdots & \varphi_{1q} \\ \varphi_{21} & 1 & \cdots & \varphi_{2q} \\ \vdots & \vdots & & \vdots \\ \varphi_{q1} & \varphi_{q2} & \cdots & 1 \end{bmatrix} \qquad (21)$$

If the network contains no rectifiers, then $\phi_{ij} = \phi_{ji}$, and the structural matrix will be symmetrical with respect to the principal diagonal. In this case the number of the elements in the network will be equal the number of elements in the matrix, located on one side of this diagonal.

One can write analogously the structure of a relay network. Since in relay network where the relays have several windings each it is important to know the direction of the current in the individual windings, we agree to write down the symbol of a winding in the corresponding cell without an arrow, if the start of the winding is connected to the node from which the circuit is traced, and we shall use the same symbol with an arrow, if the end of the winding is connected to the same node, i.e., if a winding A^k is connected to the nodes numbered α and β, and the start of the winding is connected to the node α, then

$$\varphi_{\alpha\beta} = A^k$$

and

$$\varphi_{\beta\alpha} = \overset{\leftarrow}{A}{}^k.$$

Thus, the networks of Fig. 11 can be written down in the form of the following matrices.

$$
\begin{bmatrix}
1 & 0 & aX^1 & a \\
0 & 1 & 0 & Y \\
aX^1 & 0 & 1 & \delta X^3 \\
\bar{a} & Y & \delta X^3 & 1
\end{bmatrix}
\qquad
\begin{bmatrix}
1 & 0 & \bar{a} & a & 0 & 0 \\
0 & 1 & 0 & 0 & e & \bar{e} \\
\bar{a} & 0 & 1 & 0 & \bar{b}a & \delta a \\
a & 0 & 0 & 1 & \bar{b}a & \delta a \\
0 & e & \bar{b}a & \delta a & 1 & 0 \\
0 & \bar{e} & \delta a & \delta a & 0 & 1
\end{bmatrix}
\begin{array}{l}
\bar{b}\,c \\[4pt]
\bar{b}_a\ \ell\bar{f} \\[4pt]
\bar{b}\,\bar{e} \\[4pt]
b\,\bar{e}
\end{array}
$$

We note that   by adding supplementary nodes in circuits of class Π, the form of the matrix is changed. However, since these matrices represent one and the same network, we shall call them equal. Thus, for example, if we introduce in the network of Fig. 11a additional nodes 5 and 6, we obtain the following matrix.

$$
\begin{bmatrix}
1 & 0 & 0 & \bar{a} & a & 0 \\
0 & 1 & e & \bar{f} & 0 & 0 \\
0 & e & 1 & 0 & \bar{X}^1 & \delta \\
\bar{a} & Y & 0 & 1 & 0 & X^1 \\
a & 0 & X^1 & 0 & 1 & 0 \\
0 & 0 & \delta & \bar{X}^2 & 0 & 1
\end{bmatrix}
$$

To designate networks of class H, we can use also other methods, for example the use of symbols for multiple-pole parallel or series connection /16/, or characteristic functions /31/.

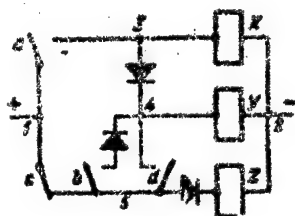## 2. NOTATION FOR STRUCTURAL NETWORKS WITH RECTIFIERS

In the presence of rectifiers in a relay network, with admittances that depend on the direction of the current and which can be assumed, in view of their low value compared with the admittances of the relay windings, to be considered . zero when the rectifier is "cut off" and unity when the rectifier "conducts," it is also very convenient to use the matrix notation, since each element of the matrix determines the admittance in one direction. If a rectifier is included in the circuit between nodes α and β, then the structural admittance from α to β will not be equal to the admittance from β to α, as can be seen, for example, from Table 5.

Table 5

Circuit	$y_{\alpha\beta}$	$y_{\beta\alpha}$
$\alpha \longrightarrow\!\!\!\!\!\Vert\!\longrightarrow \beta$	1	0
	0	1
	a	0
	1	a
	a	0
	$a(b+c)$	ac

As a result, the structural matrix for a network with rectifiers will be asymmetrical with respect to the principal diagonal. Thus, the matrix for the network of Fig. 12 will be

$$
\begin{bmatrix}
1 & 0 & a & 0 & c \\
0 & 1 & \overleftarrow{X} & \overleftarrow{Y} & 0 \\
a & X & 1 & 1 & 0 \\
0 & Y & 0 & 1 & d \\
\overline{c} & Z & 0 & b+d & 1
\end{bmatrix}
$$

To   designate the network in the form of structural formulas, the rectifiers must be assigned some sort of symbol, (M. A. Gavrilov /16/ uses the symbol k_i).

— x —

3. NOTATION FOR THE OPERATING CONDITIONS OF CONTACT NETWORKS

In formulating the operating conditions of individual contact circuits, one usually says that a given circuit should be or can be closed at definite states of the relays. The particular case when this circuit acts on some relay or some other device, one can say whether this relay operates or does not operate   for given states of the circuit elements.

Let us consider, for example, a control network with three objects, in which a signal W must appear if simultaneously not less than two control relays, A, B, or C, operate. Thus, the conditions for the appearance of the signal W can be formulated in the following manner. The circuit of the signal should be closed when relays A and B operate but relay C does not, or when relays A and C operate but relay B does not, or when relays B and C operate and relay A does not, or finally, when all three relays operate.

Table 6

Contacts pole			Lamp		number of state
A	(B)	(C)	W	X	
–	–	–	0	0	0
+	–	–	0	0	1
–	+	–	0	1	2
+	+	–	1	~	3
–	–	+	0	0	4
+	–	+	1	1	5
–	+	+	1	0	6
+	+	+	1	~	7

1) States of the relays, 2) circuits, 3) number of state.

In practice one encounters frequently cases when it is immaterial under certain states of the relays of the network whether a given circuit is opened or closed. Under these conditions, in verbal formulations, one uses the words "can be closed" or "can operate." In particular, in such "indifferent" states may be included those combinations of states of the network, which are not encountered during its operating process, i.e., so-called unused states /18, 48/.

Let us specify for example, that in the preceding case the signal relay Z should operate when relays A and C operate but relay B does not, or when relay B operates and relays A and C do not, or can operate when all three relays operate, or when relays A and B operate but relay C does not.

These states of the system, at which the given circuit must be closed, we shall call obligatory, while those at which it can be closed will be called conditional.

The operating conditions can also be written in the form of a table of correspondence, in which for each combination of states of the circuit elements there are indicated the required states of the individual circuits.

We shall agree to make up a table of correspondence as shown for our particular example in Table 6, where the non-working states of the relay are noted by minus signs, and the working ones by plus signs. As to the circuits, we shall designate by unity states in which the circuit must be closed, by zero the states in which

they must be opened, and by a tilde ($\sim$) the indifferent states.

If we now number all the states of the network (see Chapter 1, Section 2), then each circuit can be written in the form of an assembly of numbers of those states, in which the circuits should be or can be closed.

If in Table 6 relay A is assigned a weight 1, relay B a weight 2, and relay C a weight 4, we obtain the numbers as indicated in the same table.

We shall agree to write down the assemblies for each circuit in curly brackets, and include the numbers corresponding to the conditional states, in round parentheses. For the preceding example we obtain the following assemblies:

$$ f_w = \{3, 5, 6, 7\}; \quad f_z = \{2,5, (3, 7)\}. \tag{2.2a} $$

Inasmuch as the numbers of the states depend on the weights assigned to the relays in the network, in those cases when it is necessary to do so, we shall designate by means of an index for the curly brackets the relays included in the network in order of decreasing weights.

Thus, Eqs. (2.2a) are written in the form (operations with such assemblies will be discussed in detail in Chapter 6).

$$ f_w = \{3, 5, 6, 7\}_{BBA}, $$
$$ f_z = \{2, 5, (3, 7)\}_{BBA}. \tag{2.2b} $$

Numbers corresponding to obligatory states we shall call obligatory, and numbers corresponding to conditional states will be called conditional.

In the general form, when there are r obligatory states and s conditional states we obtain

$$I = \{\eta_1, \eta_2, \ldots, \eta_r, (\mu_1, \mu_2, \ldots, \mu_s)\}_B = \{N, (M)\}_B \qquad (2.3)$$

where: η_i — number of obligatory states;

N — assembly of obligatory numbers;

μ_j — number of conditional states;

M — assembly of conditional numbers;

B — base.

Finally, the operating conditions of the network can also be written in the form of algebraic formulas.

In formulating the operating conditions of any circuit, we usually use the conjunctions "and" and "or." Thus, for example, if the circuit of relay X must be closed, when relays A and C operate, it is easy to verify that the closing (normally opened) contacts of relays A and β (Fig. 13a) must be connected in series in the circuit of relay X. To the contrary, the condition that the
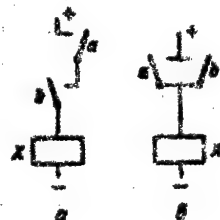

Fig. 13.

relay X must operate when relays A or B operate, leads to a parallel connection of the same contacts (Fig. 13b). Analogously, if we say that the circuit must be closed when the relay does not operate, this circuit must contain an opening (normally closed) contact of the relay.

We see therefore that the conjunction "and" corresponds to a series connection of contacts, or, in the symbolism adopted earlier, a multiplication sign (.), while the conjunction "or" corresponds to a parallel connection, i.e., to the sign of addition (+). Such a symbolism corresponds to the symbolism of algebraic logic (see Chapter 4, Section 1).

Using the symbolism employed earlier for   des-
ignating the structure of contact networks, we can write down also the action of individual circuits of the network in the form of formulas, which give the dependence of this circuit on the states of the individual contacts:

$$f_X = f(a, \bar{a}, b, b, \ldots, n, n). \qquad (2.4)$$

Thus, for relay W we obtain from the preceding example that the operating conditions   are written in the form

$$f_w = a\bar{b}\bar{a} + a\bar{b}a + \bar{a}ba + aba.$$

However, to write down the circuit Z in this manner is no longer possible, since its operating conditions include conditional states. In order to be able to represent conditional states in the formula, we introduce an additional symbol $\frac{x}{0}$, which indicates that one can take any of the expressions standing on both sides of the

bar. (The physical meaning of this symbol and operations with it will be given below in Chapter 5.)

Taking this symbolism into account, the operating conditions for the circuit Z in the foregoing example will be written

$$f_Z = \bar{a}\bar{b}\bar{n} + a\bar{b}\bar{n} + \frac{a\bar{b}n}{0} + \frac{a b n}{0}$$

Thus, the operating conditions can be written in the general form as follows:

$$f = \sum_{i=1}^{l} k_{\eta_i} + \sum_{j=1}^{k} \frac{k_{\mu_j}}{0}. \tag{2.5}$$

where k_{η_i} and k_{μ_j} are the constituents of the obligatory and of the conditional states with respect to the numbers η_i and μ_j.

In some cases, particularly for $n > 4$, it may be convenient to use a "coordinate" notation, used for example by O. Plechl [20] and A. Svoboda [55 — 56]. In this notation one draws a rectangular grid with 2^n cells, corresponding to 2^n constituents. The variables are broken up into two groups (when n is even — the groups are equal), and the combinations of the variables in each group form the "coordinates" of the rows and the columns, and each constituent is determined by the product of these "coordinates." The presence or absence of constituents in the formula is noted by placing either one or zero in the corresponding cells (O. Plechl used the symbols infinity and zero, while A. Svoboda used in some cases a dot instead of one and leaves the cells correspond-

ing to the missing constituents empty). The cells containing condi-
(½)
tional constituents are marked by a $\sim$ or one-half (Plechl uses
(!)
a question mark).

Thus, we can write down

$$I = \{1,7,8,9,10,12,(0,6,11,13,14,15)\}_{\textit{FDBA}}$$

	6 a	6 ʙ	6 c	6 ɴ
c ʙ	$\sim$	1	0	0
c ʙ	0	0	$\sim$	1
e ʙ	1	1	1	$\sim$
z ʙ	1	$\sim$	$\sim$	$\sim$

(2.6)

If we now assign to the "coordinates" weights in accordance
with the weights assigned to the individual variables, then each
cell will correspond to a constituent with a number equal to the
sum of the weights of the "coordinates." It is most con-
venient to arrange the variables in such a way that the "coordi-
nates" of the columns are $0, 1, 2, \ldots, (2^{\ell} - 1)$ while those of
the rows are $0.2^{\ell}, 2.2^{\ell}, 3.2^{\ell}, \ldots, (2^{n-1} - 1).2^{\ell}$, where ℓ is the

	0	1	2	3
0	0	1	2	3
4	4	5	6	7
8	8	9	10	11
12	12	13	14	15

number of columns used in the "coordinates."

For example (2.6) we obtain the following numbering for the cells (constituents).

	0	1	2	3
a	~	1	0	0
b	0	0	~	1
c	1	1	1	~
d	1	~	~	~

In the coordinate notation each row and column can be assigned letter or number symbols and then each constituent (each vertex) can be denoted briefly by a combination of these symbols. For example, if we number the rows of (2.6) with letters and the columns with numbers, then the constituent k_1 assumes a designation a1, constituent k_7 becomes b3, etc. The symbol ac1 will correspond to the sum of two neighboring constituents k_1 and k_9, i.e., to the expression $a\overline{b}c$, while b2.3 will correspond to the sum of k_6 and k_7, i.e., to $bc\overline{d}$. The symbol bd 2.3 corresponds to the sum of four constituents k_6, k_7, k_{14}, and k_{15}, i.e., to bc, etc. These symbols are used in particular by A. Svoboda to   abbreviate the notation.

We shall call the designation of the necessary and possible conditions for the circuit in the form (2.3), (2.5), (2.6) or some other form, the general solution for this circuit. The general solution contains all the possible versions of the particular conditions or particular solutions of circuits, which can differ in the presence or absence of individual conditional terms, but which satisfy the specified conditions. It is easy to verify that in the presence of s conditional constituents k_μ in the general solution, the number of different particular solutions will be 2^s /57/.

The operating conditions of a contact q-terminal network, for specified circuits f_{ij} between all nodes (i = 1, 2, ..., q and j = 1, 2, ... q), can be written in the form of a characteristic matrix or a matrix of total admittances

$$ M = \begin{bmatrix} 1 & f_{12} & \cdots & f_{1q} \\ f_{21} & 1 & \cdots & f_{2q} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ f_{q1} & f_{q2} & \cdots & 1 \end{bmatrix} $$

(2.7)

It must be noted that for a q-terminal network not all the admittances f_{ij} can be specified arbitrarily, since several circuits will appear between the poles i and j through other poles. As shown by A.G. Lunts /31/, in order for a matrix M to be a characteristic matrix, it is necessary and sufficient to have

$$M^2 = M, \qquad (2.8)$$

i.e., that the squaring of the matrix (in the sense of Boole) leads to the same matrix.

If this is not satisfied, then the conditions designated by the matrix M cannot be realized by a contact network.

It must be noted, however, that in most practical cases it is necessary to specify the circuits between all the poles, and therefore the use of the notation for the conditions in the form of matrices is very limited.

4. NOTATION FOR THE OPERATING CONDITIONS OF MULTIPLE-STEP NETWORKS

In multiple-step /sequential/ networks, ... the primary importance attaches to the sequence of the arrival of the external signals and of the operation of the actuating circuits, and also the instants in which the intermediate relays operate or drop out.

In the formulation of the operating conditions of the network one usually says that the ... closing or opening of a certain circuit or the operation or drop-out of a certain relay is followed by the operation (release) of a second relay or by the closing (opening) of some actuating circuit. A distinction is made here between the operation of the receiving elements, which are operated by external signals, and the operation of the intermediate relays, which operate as a rule in local circuits.

Thus, for example, for the circuit of the binary pulse divider
with two intermediate relays A and B, operated by the con-
tacts of the receiving pulsed relay P, the operating conditions are
formulated as follows: when pulse relay P operates for the first
time, relay A operates. After relay P drops out, relay B operates.
During the second operation of the relay P, relay A drops out and
after dropping out of the relay P, relay B drops out. The operating
cycle of the network terminates and the network returns to the
initial state. The actuating circuit X should be closed during the
second pulse and can be closed during the interval between the first
and second pulses.

The operating sequence of a multiple-step network can be
represented by a relay-operation graph (Fig. 14) consisting of
several stages of the network operation, separated by different
external signals (in this case instants of start and end of the
incoming pulses).

The graph, however, does not give a complete picture of the
interaction between the individual network elements and therefore
the most convenient form for the notation of the operating conditions
of a multiple-step network is the so-called connection table, con-
sisting of steps, i.e., periods during which the receiving and
intermediate elements (relays, keys, etc.) of the network do not
change their states. The connection table for each element, as well
has
as for each actuating circuit, a line in which are noted the states

of the elements and the circuits during each step. The operating states of the receiving elements which respond to external signals are denoted by a heavy line. In the lines for the intermediate relays those steps in which conditions have been created for the operation of the given relay (but the relay has not yet operated) are designated by an arrow ($\longrightarrow$); steps in which the relay operates are designated by a plus (+), and   steps during which conditions are created for its release (but it is still pulled in) are designated ($\dashv$). For the actuating circuits, a solid line designates the periods when these circuits should be closed, and a dotted line designates when they may be closed.

The connection table for a binary pulse divider is shown in Fig. 15.

The changeover from one step to the next is due to a change in the state (operation or drop-out) of one of the relays of the circuit. This change, in turn, is made possible either by a change in the external signals, or by the creation of corresponding conditions for the intermediate relays inside the network. In the former case the steps will be stable, i.e., their duration will be determined by the duration of the external signal, which can be arbitrary and independent of the given network.

In the second case, the steps will be unstable, since their duration is determined by the operating or drop-out time of the relays, and the table must have in these steps either a $\longrightarrow$ sign

Fig. 14.
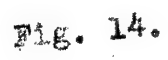
No of state

Fig. 15.

Fig. 16.

for a minus sign.

All these steps can be numbered, for example, in the order of their sequence. It is more convenient, however, to use here the same system of numbering as in the correspondence tables (Chapter 1, Section 2).

Fig. 15 shows the numbering for the cases when the relay P is assigned a weight 1, relay A a weight 2, and relay B a weight 4. In this case the steps numbered 0, 3, 6, and 5 are stable.

There exists also other methods of writing down connection tables. Thus, for example, Doctor of Technical Sciences M. A. Gavrilov /16/ recommends that the connection tables be written in a somewhat different form, without separation into receiving and intermediate elements, as shown in Fig. 6d.

Table 7

Inputs $(x_1 x_2)$				outputs
00	01	10	11	
①	2	3	10	00
⑥	②	9	④	01
⑦	8	③	⑤	10
1	⑧	⑨	⑩	00

D. Huffman /95, 96/ gives another tabular method of writing down operating conditions of multiple-step networks.

In his table he places on the horizontal lines different combinations of the states of the inputs (receiving elements), and on the vertical he arranges the states of the outputs (actuating circuits). At the intersection are placed the serial numbers of the states of the network, and if this state is stable the number is enclosed in a circle. A change in the states of the inputs leads to a change in the number of the state along the horizontal line. If this number is not encircled, this indicates that the circuit should change as a result of the change in the states of the intermediate relays, along the vertical into a stable state with the same number.

Table 7 is compiled by the Huffman method, starting with the following conditions.

In the initial state of the network, not one of the two receiving relays X_1 and X_2 operate. When one of these relays operates, the corresponding actuating circuit is closed and the state of the actuating relays should remain the same until a state is produced in which only   the second relay operates. Now the actuating circuit should be   opened, and the circuit should return to the initial state only after both receiving relays are disconnected. The presence of several circuit numbers along the same column indicates the need of introducing intermediate relays into the

network.

This method of notation may be convenient when the input signals are received by several inputs and arbitrary sequences of the signals are possible, i.e., when for any state of the network any signal can be received. In this case it is necessary to construct a large number of connection tables for different sequences. As regards telephone control systems, such conditions are not encountered there in practice, and the number of possible sequences of external signals is usually limited.

In addition to tabular methods, there exist other methods of writing down the operating conditions of sequential networks. Thus, for example, a record can be made in the form of the so-called connection formulas /16/, or in the form of a sequence of numbers of states of the network /99/. The operating conditions of the network can also be represented in the form of transition diagrams and by other methods /60, 61, 114, 117, 118/.

For synthesis purposes, the most convenient is a notation in the form of connection tables, since, as will be shown below, such a notation is clear and makes it possible to determine simply the number of intermediate relays and to proceed to the construction of the structure of the networks.

5. CHANGE FROM CONNECTION TABLES TO CORRESPONDENCE TABLES

The connection table in the form shown in Fig. 15 not only

shows the　　　　operating sequences of the individual elements and circuits of the network, but makes it also possible to establish in what states �ᵒᶠ the network the actuating relays should and may be closed, and also specify conditions for operation of the intermediate relays. In other words, we can change over from the connection table to a correspondence table, while the latter should also contain the inter-mediate-relay circuits. These circuits should be closed in all steps, in which the symbols ⟶ and ╈ . are designated for the relay. Thus, the connection table of Fig. 15 leads to Table 8.

Table 8

3) N состояния	1) Реле			2) Цепи		
	P	A	B	f_A	t_b	t_x
0	−	−	−	0	0	0
1	+	−	−	1	0	0
2	−	+	−	1	1	~0
3	+	+	−	1	0	0
4	−	−	+	0	0	~1
5	+	−	+	0	1	1
6	−	+	+	1	1	1
7	+	+	+	0	1	1

1) Relays, 2) circuits, 3) number of states.

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Chapter 3

SYNTHESIS OF SEQUENTIAL NETWORKS

As already indicated, the principal relay networks in telephony are sequential networks. The construction of a sequential network is based primarily on the conditions for the interaction between the network and other instruments, devices, and networks, i.e., the sequence of the external signals fed to the given network and the requirements on the signals produced by the network.

In addition to these basic conditions, one may specify also other supplementary conditions. Thus, for example, a limitation may be imposed on the number of springs in any particular relay, on the number of incoming or outgoing wires, etc. Requirements may also be specified for the operation of the network under definite types of faults or distortion in the incoming signals. All this can influence the network to a considerable extent.

The problem of structural synthesis of a sequential network includes the following:

a) Formulation and notation (for example, in the form of a connection table) for the operating conditions of the projected network.

b) Subdivision of the network into functional blocks (if the operating conditions make it necessary) and determination of the number of intermediate relays which must be introduced into the network to insure a normal dependence of the states of the actuating circuits on the states of the receiving elements.

c) Compilation of the structure of the network.

In the present book we consider the construction of networks with a minimum number of relays and other elements.

2. COMPILATION OF CONNECTION TABLES

The most convenient form for writing down the operating conditions of a sequential network are connection tables (see Chapter 2, Section 4).

The process of compiling connection tables reduces to the following.

1. One determines the skeleton of the designed network with an indication of the inputs and outputs, receiving elements connected with the inputs, and actuating circuits and elements connected with the outputs.

2. A preliminary table is compiled, in which one enters only the operation of the receiving elements (or the sequence of the external signals) and actuating circuits, i.e., one records essentially the basic requirements imposed on the designed network.

If several sequences of signals are possible in the operation

of the network, these sequences should be reflected in the form of individual parts of the connection tables.

3. From an analysis of this table one determines the functional blocks in which it is necessary to break down the network and the necessary number of relays which must be introduced into the network in order to make the table realizable, and also those periods, during which the intermediate relays should change their states.

4. The table is then expanded by introducing into it the intermediate relays and steps, during which these relays change their state. After verifying for   realizability, one can proceed to a compilation of the network itself.

By way of an example of constructing a preliminary   connection table, let us consider the synthesis of a relay assembly (RSL) for two-way connecting line /119/ between stations TsBx3x2 and TsBx2 (a complete synthesis of the network is given in /18/, pp 148 — 153).

Fig. 1   shows a skeleton diagram of the set RSL, which includes the incoming line wires a and b, jack J, receiving relay L (to receive the call from the line), P, and Y, which insure interaction with the relay of the cord pair of the TsBx3x2 switchboard, and the following actuating elements: the busy blinker BB and the calling lamp CL. The same figure shows   some of the devices in the cord pair, interacting with the RSL: plug P_f, sign-off relays OP_1 and OP_3, and the contact of the sign-off relay OP_4,

which shunts the 400- ohm resistance when a sign-off signal arrives from the other subscriber. Relay OP_1 operates only when the resistance of its circuit does not exceed 600 ohms, while relay Y operates only at a circuit resistance up to 200 ohms. Relays P and OP_3 operate in all cases when their circuits are closed. In accordance with the requirements, the following additional circuits should be produced in the network: that of the calling lamps (f_{CL}), the busy blinker (f_3), the grounding of a wire $(b(f_b))$, for the transmission of the response signal to the RTS, and for the shunting of the 2000-ohm resistance (f_{sh}), to control the relay OP_1 in the cord pair.

The entire process of network operation consists of 11 basic stages, listed in the preliminary connection table (graph of the operation of the receiving and actuating elements) on Fig. 18.

0) network not operating; 1) receipt or call from the TsBx2 station; 2) interrogation (the interrogation plug is inserted in the jack); 3) establishment of the connection with the line of the called subscriber; 4) connection after the called subscriber or station TsBx2 answers; 5) sign-off on the part of the local subscriber and transmission of the sign-off to the TsBx2 station; 6) disconnection on the TsBx3x2 switchboard (plug removed from jack); 7) sign-off on the part of the TsBx2 station after sign-off on the part of the local subscriber; 8) freeing of the line; 9) engagement of the line on the part of the TsBx3x2 station; 10) sign-off on the part of the TsBx2 station ahead of the
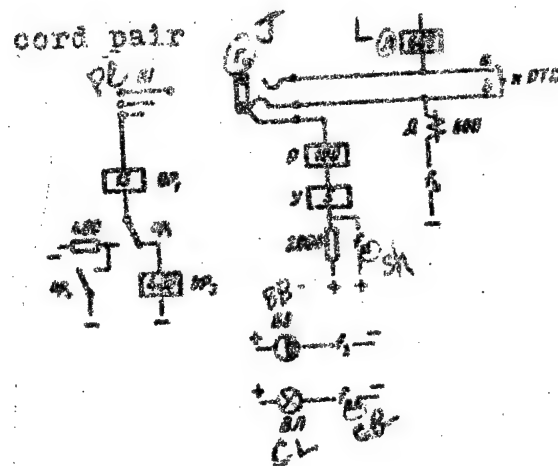
cord pair

Fig. 17.

	Stages	a								б			в				
		0	1	2	3	4	5	6	8	(5)	7	8	0	9	4	10	8
2) Receiving relays	(n)																
	p																
	y																
3) Actuating relays	c-вл																
	вз/вз																
	b																
	sh.ш																

Fig. 18.

sign-off on the part of the local subscriber.

The actions may occur in different sequences. Thus, the line may be engaged both on the side of the TsBx2 station (Fig. 18a) as well as on the side of the switchboard (Fig. 18c).

Analogously, sign-off can be on the part of the subscriber of the switchboard (Fig. 18a, b) or on the side of the TsBx2 station (Fig. 18c). All these possibilities should be noted on the connection table.

3. CONDITIONS OF REALIZABILITY OF THE TABLE

In order for the table of connections to be realizable in the form of a relay network, it is necessary on the one hand to insure the remembering of the sequence of the signals, and on the other hand, to produce the actuating circuits during the corresponding steps.

On the basis of the analysis of the action of the relay networks, the following realizabili of conditions have been formulated /18, 49/:

1) Different states of the actuating circuits should correspond to different states of the receiving and intermediate elements of the network.

2) If one encounters in the operation of the network steps with identical states of the elements and if at least one such step is unstable, all these steps should be unstable, and in the

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next steps the states of the elements should be the same.

If any of these conditions is violated, it is necessary to introduce into the network one or several intermediate relays or to change in a corresponding manner their   operating sequence.

It is necessary to bear in mind here that if in any step several relays change their states simultaneously, intermediate states may appear due to simultaneous change in the states of these relays. Unstable steps corresponding to these intermediate states must be taken into account when verifying the correctness of the operation of the network.

Starting with the first realizability condition, the intermediate relays should be introduced in such a way that one can distinguish those states of the network, in which the state of the receiving elements is the same, but the state of the actuating circuits is different.

Thus, if there are two stages with identical states α_1 and α_2 of the receiving elements with different actuating circuits, separated by stage β (Fig. 19a), one must introduce an intermediate relay, which changes its state first during the stage β, and a second time beyond one of the stages α.

If on the other hand there exist two   pairs of states $\alpha_1 - \alpha_2$ and $\beta_1 - \beta_2$, which must be distinguished (Fig. 19b), then the intermediate relay must change its state during the
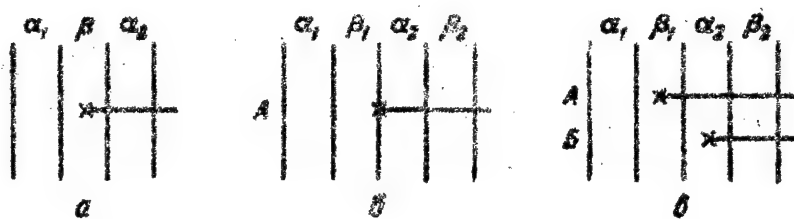
Fig. 19.

time of transition from state β_1 to α_2. However, it is impossible to realize such a construction of the network, since the second condition of realizability will not be satisfied. Actually, if the intermediate relay A is set to operate at stage α_2, then an unstable step with state α_2 of the receiving relays appears. However, the same state is observed at the stage α_1 and consequently, relay A will operate already at this stage. Therefore for the sequence of the states of the relay as represented in Fig. 19b, it is necessary to introduce two intermediate relays A and B, of which one will change its state at the stage β_1 and the second at the stage α_2, as shown in Fig. 19c.

It becomes necessary also to distinguish between periods with identical states in those cases when in these states the actuating circuits are identical, but the sequence of these states is of importance for the change of any actuating circuit. Thus, for example, if a counting network must close the circuit only after a definite number of incoming pulses, then all the intermediate states of the network should be different, so as to be able to say how many pulses have already been received.

4. DETERMINATION OF THE NUMBER OF INTERMEDIATE RELAYS

An analysis of the connection table on the basis of the realizability conditions makes it possible to determine the number of relays which must be introduced into the table, in order to make

it realizable, and also those periods in which these relays must change their states.

As has been shown by investigations carried out by V.G. Lazarev /92 — 94/, the number of relays which must be introduced depends on the maximum number m_{max} of identical states ("coincidences," in the terminology of V.G. Lazarev) of the receiving elements, in which the state of the actuating circuits is different, or else these states are essential for the realization of the specified sequence. The smallest number s of the intermediate relays is obtained in this case from the following inequalities:

$$2^{s_1} \geqslant m_{max}. \tag{3.1}$$
$$2^{s_2} \geqslant 2m_{max} - 1. \tag{3.2}$$

The first of these inequalities gives a minimum value, but the network with this number of intermediate relays can be realized only when between these identical states of the intermediate relays, which must be distinguished, there exists states which need not be distinguished. In the opposite case it becomes necessary to use inequality (3.2), which gives one additional relay.

Thus, for example, for the connection table (Fig. 18) we, see that identical states of the receiving relays exists at stages 1 — 6, 2 — 4, 3 — 5, and 9 — 10, whereas the state of the actuating circuits should be different in these stages (with the exception of stages 9 and 10).

We thus obtain $m_{max} = 2$, hence $s_1 = 1$ and $s_2 = 2$. In order

Fig. 20.

to distinguish the states of the relay, it is enough to introduce one relay, which changes the state between stages 3 and 4 and after the sixth stage. However, to insure operation of the relay in stage 4 after the drop-out of relay Y is impossible, since the second realizability condition will not be satisfied. It is therefore necessary to introduce into the network two intermediate relays.

Fig. 20 shows the total table of connections obtained for this case with a breakdown of the possible variants of network operation: d -- on the switchboard TsBx3x2 the disconnect takes place without establishing a connection with the subscriber; e -- sign-off when the called subscriber does not answer; f -- sign-off on the part of the TsBx2 station prior to sign-off by the switchboard subscriber; g -- the same in the case of failure of the called switchboard subscriber to answer. The same table indicates the number of the states for each step with allowance for the transition steps, which can occur as a result of non-simultaneous drop-out of relays P and Y. States with numbers 4, 5, 6, 7, 12, 13, 14, 24, and 28 are not used.

5. Choice of Connection Sequence for the Intermediate Relays

Following the determination of the number s of intermediate relays, which must be introduced in the network, it is necessary to find the sequence of operation of these relays. For this purpose we note first those stages, in which the intermediate relays

should change their states. In some cases this may be not a single
stage, but several stages, i.e., the period during which the state
of the intermediate relays should change. Thus, for example, for the
sequence shown in Fig. 21, the intermediate relays should change
their states at stages α_1 or β_1 (to separate γ_2 from γ_3), γ_2
(to separate $\alpha_1 - \beta_1$ from $\alpha_2 - \beta_2$) and β_2 to separate
γ_2 from γ_3), and finally ahead of γ_1 or α_2.

We shall agree to call these stages of the operation of the
network, during which the intermediate relays must change their
states, the instants (or periods) of connection.

Thus, one obtains in the table m "obligatory" instants (or
periods) of connection. Since each relay should change during the
the state of
cycle an even number of times, then, in the case of m odd, one
must of necessity note one instant of connection, which, from the
point of view of realizability of the network, can be chosen ar-
bitrarily (and may also coincide with any one of the obligatory
instants). One can choose equally arbitrarily any number of
instants of connections from among $2^s - m$, in the case when m
$< 2^s - 1$, where 2^s is the maximum number of changes of the states,
which can be obtained for s intermediate relays.

After the obligatory instants of connection have been deter-
mined, and the additional number of such instants which can be
created is ascertained, it is necessary to choose the operating
sequence with intermediate relays and to add them to the connec-

tion table. In most cases this problem is solved in many ways. The ambiguity is due on one hand by the aforementioned arbitrariness in the choice of several instants of connection, and on the other hand by the fact that at ... specified instants of connection and at a specified number of relays, different sequences can be chosen.

Thus, for example, in the case of two intermediate relays (s = 2) and four instants of connection (m = 4), one can obtain only one sequence (if one disregards the case of interchange of relays), as shown on Fig. 22a. In the case of three relays and eight points of connection (s = 3, m = 8) there can be made up two different sequences (Fig. 22b) which are mutually reversible. For four relays (s = 4, n = 16), there are nine different sequences (including the inverse ones) /120/, three of which are shown in Fig. 22c. Upon further increase in the number of the relays, the number of sequences increases sharply.

If $m < 2^s$, the number of possible sequences increases.

The choice of any particular operating sequence for the intermediate relays manifest itself in the complexity of the network, since the number of contacts changes both in the intermediate-relay circuits, and in the actuating circuits. This in turn may render the network non-realizable because of the lack of springs on the relay and will make it necessary to increase the number of relays in the network.

There is still no general rule for the choice of operating
the
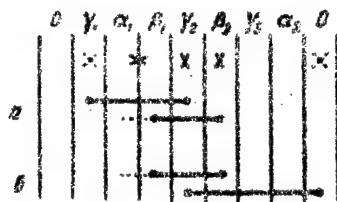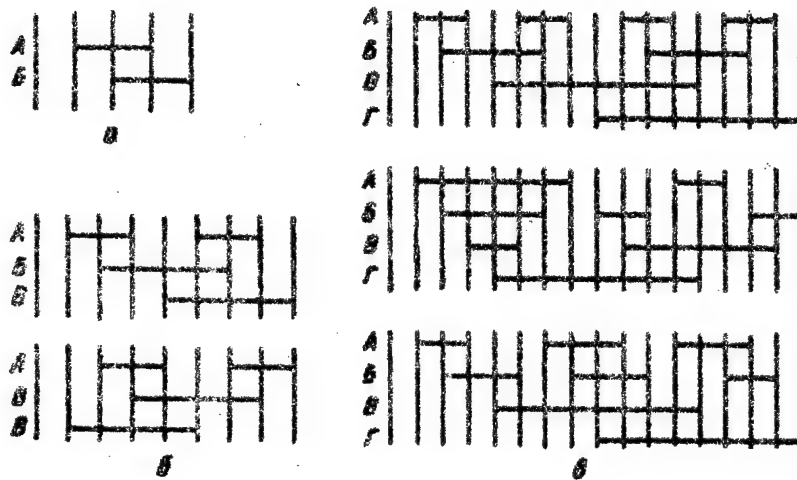sequence of relays. In this book we make an attempt to

Fig. 21.

Fig. 22.

suggest several recommendations on this matter, primarily from the point of view of obtaining networks with a minimum number of contacts on the intermediate relays.

Experience in the design of relay networks shows that as the number of relays in a sequential network increases, where the network realizes all 2^n states which can be produced by the n relays contained in the network, the number of contacts increases much more rapidly than the number of steps produced by these relays.

Actually, in a network that realizes all the 2^n states, the action of each relay depends on all the remaining relays, and consequently, in the circuit of each relay there should be included contacts of all other relays. Since in a network having n relays the number of operations and drop-outs of all these relays is 2^n, during the entire operating cycle there will be produced $n2^n$ contact circuits.

As a result, it is impossible to construct in general networks with five relays of the RPN type, which would realize all the 32 states. This leads to the conclusion, thus far not confirmed theoretically, that circuits with five and more relays must as a rule be broken down into individual blocks, operating by functional features or cyclically, when the operation of one block (cascade) depends on the preceding block (cascade). In either case it becomes impossible to use all 2^n states, since some of them should be used for the transmission of signals from one block to the other.

Consequently, it may be found necessary to use more relays than given by inequalities (3.1) and (3.2).

In the operation of the intermediate relays, when they are all broken down into several cascades, and the receiving elements act only on the relays of the first cascade, while the relays of the succeeding cascades count the periods (cycles) of operation of the preceding cascades, the total number M of the connection instants, which must be realized by the network in the case of a cascaded structure, depends on the cascades into which the network is broken down and how one cascade acts on the succeeding ones.

Without stopping in detail for derivations, we give the formulas for the determination of the maximum number M of the instants of connection, which can be obtained with a network containing s intermediate relays, broken up into k cascades, such that

$$s_1 + s_2 + \ldots + s_k = s. \qquad (3.3)$$

We consider here only the following basic methods of action of the preceding cascades on the relay of the following cascades, and we shall assume that there is no inverse reaction between cascades.

1. The action is transmitted only from one relay of the preceding cascade, which operates only once during the operating cycle of this cascade.

$$M_1 = 2^{s-k+1}. \qquad (3.4)$$

If the network is constructed such that the last stages of the cascade are retained still during the period when the acting

relay of the preceding cascade has not yet changed its state, the number M increases somewhat

$$M_2 = 2^{s-2+1}(1 + 2^{-s_k}).$$ (3.5)

2. The signal is transmitted from all relays of the preceding cascade:

$$M_3 = 2^{s_k} \prod_{i=1}^{k-1} (2^{s_i} - 1).$$ (3.6)

In the particular case when $s_1 = s_2 = \dots = s_k = s/k$ we have

$$M_3' = (2^{s_i} - 1)^{k-1} 2^{s_i}.$$ (3.7)

3. The signal is transmitted from all the relays of all the preceding cascades:

$$M_4 = 2^{s_k} - \sum_{j=2}^{k} 2^{\sum_{i=j}^{k} s_i}.$$ (3.8)

We can see from (3.4) — (3.8) that as a rule there should be $s_i > 1$, and only in the cases (3.5), (3.6), (3.8) can one relay be employed in the last cascade, i.e., we can have here $s_k \geq 1$.

It follows from the same formulas that

$$M_1 \leqslant M_2 \leqslant M_3 \leqslant M_4,$$ (3.9)

As shown, for example, in Table 9 for $s = 5$ and 6. It should be noted that M is increased here as a rule because of the complications introduced into the contact network.
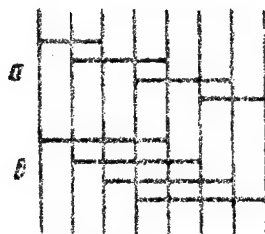
Table 9

Fig. 23.

s	k	s_i	M_1	M_2	M_3	M_4
5	1	5	32	32	32	32
	2	4,1	—	24	30	30
		3,2	16	20	28	28
		2,3	16	18	24	24
	3	2,2,1	—	6	18	22
6	1	6	64	64	84	64
	2	5,1	—	48	62	62
		4,2	32	40	60	60
		3,3	32	36	56	56
		2,4	32	34	48	48
	3	3,2,1	—	24	42	54
		2,3,1	—	24	42	46
		2,2,2	16	20	36	44

In references /104/ it has been shown that in counting networks it is advantageous to use a cascade construction, using two or three relays per cascade.

Considerable economy in the number of contacts can be obtained if the network is constructed such that each of the intermediate relays changes its state   exactly twice during the cycle of the cascade in which this relay is included, as shown, for example, in Figs. 23a and 23b. In such a sequence, s_i relays produced during one cycle two $2s_i$ steps.

Depending on the interaction between the cascades and on the number of relays in each cascade, the total number M of the instants of connection, which must be produced, is   given by the following formulas:

a) When the signal is fed to the next cascade from one relay

of the preceding cascade

$$M_b = 2s_1 s_2 \ldots s_k;$$ (3.10)

b) When the signal comes from all relays of the preceding cascade

$$M_6 = 2s_k \prod_{i=1}^{k-1} (2s_i - 1);$$ (3.11)

c) When all the signals come from all relays of the preceding cascades

$$M_7 = 2^k s_1 s_2 \ldots s_k - \sum_{i=1}^{k-1} 2^{k-i} s_{i+1} s_{i+2} \ldots s_k.$$ (3.12)

In the particular case when $s_1 = s_2 = \ldots = s_k$, we obtain

$$M_5' = 2s_1^k;$$ (3.13)

$$M_6' = 2s_1 (2s_1 - 1)^{k-1};$$ (3.14)

$$M_7' = (2s_1)^k - \frac{(2s_1)^k - 2s_1}{2s_1 - 1}.$$ (3.15)

When $s_i = 2$ we obtain the same solutions as in the preceding case of cascade action. (Networks of the last type were studied by Candidate of Technical Sciences V. I. Ivanov. He calls them simple cyclic networks and shows that to insure operation of such networks it is necessary to have nearly two or three contacts on each relay, and all relays need be of the same type /101, 102/.)

The use of cascaded operation of the intermediate relays makes it possible to reduce the number of contacts in the network, but this may make it necessary to use more intermediate relays than

given by Eqs. (3.1) and (3.2). In some cases this must be done. In the arguments given above we mentioned only contacts necessary to insure operation of the intermediate relays of the six network. The need of placing in the relays contacts for the actuating six circuits

and   the

imposes still additional requirements in the compilation of a network such contacts must be taken into account.

6. Change in the State of Several Relays Within a Single Step

When choosing the operating sequence of the relay, one may encounter a case when during one step conditions are produced for the change of state of several relays, as shown, for example, in Fig. 24a. In practice, however, the operating times and the drop-out times of relays always differ somewhat, and consequently ~~transition~~ transient steps occur, as shown in Fig. 24b and 24c. If one considers that the relative delay of each relay is a random quantity, then any possible combination of the states of these relays is possible.

In other words, if during any one step conditions are produced for a change in the states of v relays, the appearance of $2^v - 2$ different combinations is possible;   these combinations should not be used in other periods, but cannot be considered non-useable.

We see therefore that in the construction of connection tables it is necessary to avoid transients,   in which several intermediate relays must change their states simultaneously. Such a sequence is permissible only in the exceptional case, when, for

example, the circuit must be returned to its initial position.

In those cases, when the conditions for the change in the state of several relays are produced simultaneously (particularly in the case when these relays are receiving relays), and when, upon appearance of transient steps the operation of the network may become faulty, measures must be taken to insure that a definite sequence in the relay operation be observed, either making one relay dependent on the other, or by using relays with suitable time delays.

Thus, for example, in the network of Fig. 17 the relays P and Y are disconnected simultaneously when the circuit opens during the instant when the plug is removed from the jack. Here, as can be seen from the connection table of Fig. 20a, (there may be produced, during the start of stage 6a) transient states of the networks numbered 21 (relay P drops out first) or 19 (relay Y drops out first). Analogously, states number 20 or 18 can occur upon transition from the stage 7a to 8a (Fig. 20b, g) and 29 or 27 upon transition from state 3 to stage 6a (Fig. 20d). Steps number 18 and 19 are encountered during the operation of the network, and in these states the actuating circuits do not correspond to the circuits in the same steps, between which transient steps occur with the same numbers (23 -- 17 in Fig. 20a -- 22 -- 16 in Fig. 20b, g). The appearance of transient steps numbered 20, 21, 27 and 29, on the other hand, does not violate the realizability of the table. Consequently, the network should be constructed such that relay Y

drops out later than relay P. This can be obtained by making the

relay with the time delay or by

providing an additional supply to the voltage Y so that it

is retained in the local circuit through a normally open contact of

relay P.

7. CHOICE OF SEQUENCE OF OPERATION OF INTERMEDIATE RELAYS TO OBTAIN A MINIMUM OF CONTACT IN THE ACTUATING CIRCUITS

We now consider how to choose the sequence of operation of the

intermediate relays from the point of view of the minimum number of

contacts for the actuating circuits. It is quite obvious that if

one introduces into the network as many

intermediate relays as   there are actuating circuits

necessary, and if these relays are set to operate in the same se-

quence   with which the actuating circuits must be closed, then

for each of these circuits it is necessary to have one contact, i.e.,

the total number of contacts in the actuating circuits will be a

minimum. However, in this case the network as a whole may not be

optimum from the point of view of the number of relays and the total

number of contacts.

Starting with   such arguments, we can recommend that

from the point of view of reducing the number of contacts in the

actuating circuits, it is necessary to choose the instants of

connection and disconnecting of the individual intermediate relays

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such that the operating periods of these relays coincide with the

closing (or opening) periods of the individual actuating circuits, particularly those in which it is required to produce simultaneously several circuits (to close   the circuit to several lines).

In conjunction with these recommendations, which were given above, one can choose the operating sequence for the intermediate relay such that the total number of contacts will be close to a minimum.

In the investigation of counting circuits, the author has shown /103, 104/ that if a decimal counter is made with a RPN relay with 15 springs, then in the cascade construction one can obtain networks with the characteristics shown in Table 10.
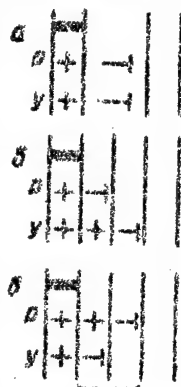
Table 10

1) Число реле	2) Число пружин в жестких цепях		5) Число контактов в каждой исполнительной цепи	6) Число пружин на 10 однопроводных выходов	7) Максимальная проводимость исполнительных цепей
	3) на 1 реле	4) общее			
5	7—8	36	3—4	27	1
6	6—7	41	2	23	2
11	5—6	63	1	20	4

Transient steps
Fig. 24.

1) Number of relays, 2) number of springs in local circuits, 3) per relay, 4) total, 5) number of contacts in each actuating circuit, 6) number of springs for each group of ten single-conductor leads, 7) maximum number of wires in the actuating circuits.

Fig. 25

1) Clamping relay 2) Counter 3) Tens 4) Units 5) Clamping relays
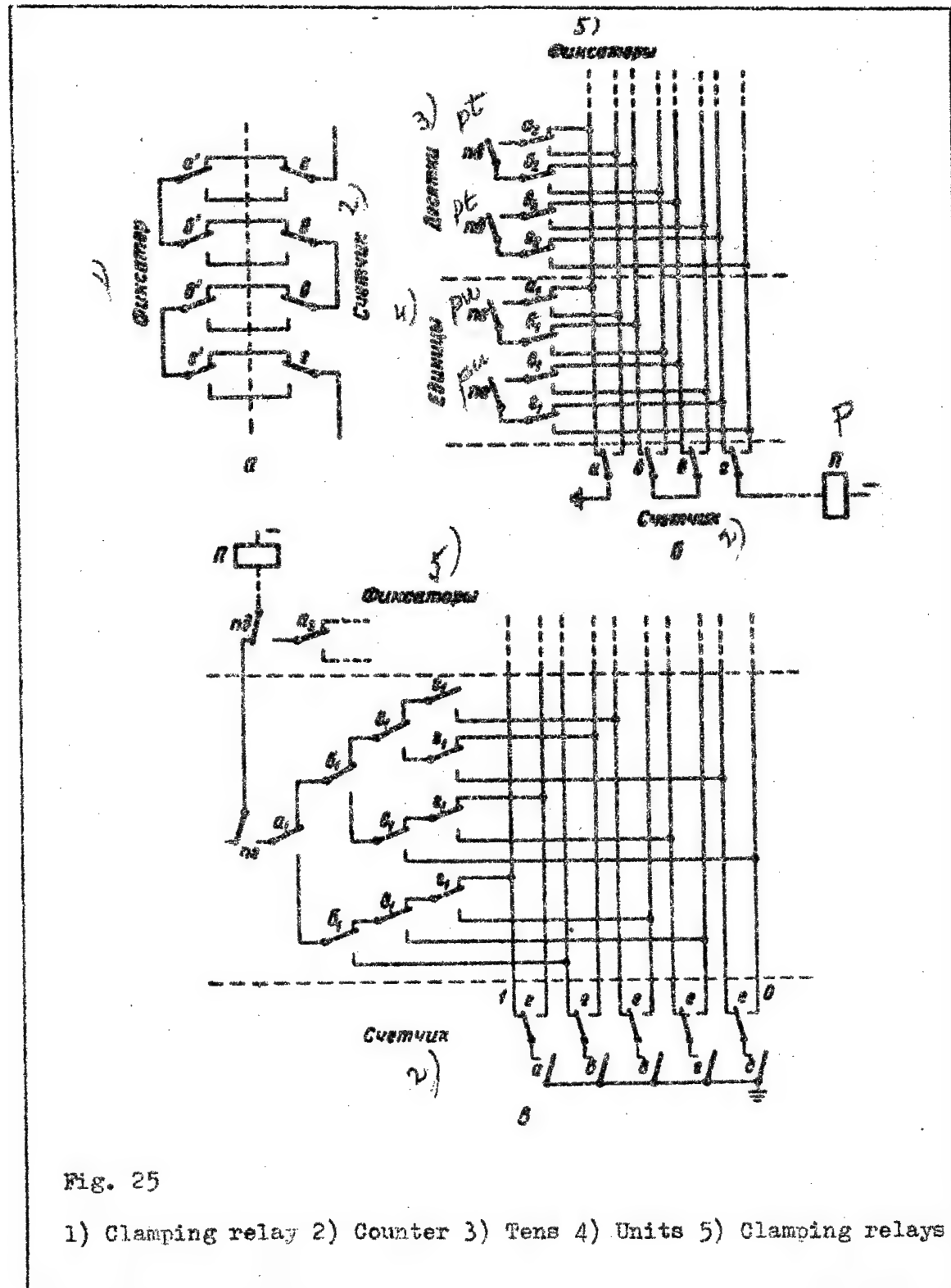
By way of another example, of simplification, we give a counting network used in key-pulsing apparatus /121/ to establish correspondence circuits after a definite number of pulses, depending on the state of the clamping relays. The greatest economy in the contacts, and sometimes also in relays, is obtained if the clamping code is matched with the counting system, i.e., if the state of the clamping relays A_1, C_1, D_1, and E_1 for each number are made the same as the state of the counting relays (A, C, D, and E) after the count of the corresponding number of pulses. In this case, to establish the correspondence circuit it is necessary to have one relay each in each double-throw contact group, connected as shown in Fig. 25a. The diagram of the correspondence circuits for the key-pulsing apparatus, constructed in accordance with this principle /18/, assumes the form shown in Fig. 25b, where a_1, b_1, c_1 and d_1 are the contacts of the relays that fix the unit numbers; a_2, b_2, c_2 and d_2 are the contacts of the relays that fix the tens numbers, etc.; pf and pd are contacts of the double-throw relays, which close respectively the circuits when unit numbers (pd) and tens (pf) are sent, etc.

In an existing key-pulsing apparatus network /121/, shown in Fig. 25c, these functions require only five relays and 88 springs.

8. WAYS OF REDUCING THE NUMBER OF INTERMEDIATE RELAYS

As seen from the analysis of the connection tables, by comparing the work of the receiving elements and of the actuating

circuits one can obtain the minimum number of intermediate relays, necessary to introduce into the network to realize the specified sequences.

In some cases, however, this number can be reduced somewhat. One of the means of reducing the number of intermediate relays is to use receiving relays as intermediate relays. Such a use is possible in the case when in accordance with the operating conditions there exist period, during which the given receiving relay does not operate, i.e., when the signal received by this relay does not arrive. In this case the given relay can be used as an intermediate, operating during these times.

The simplest method of using such a possibility is to retain the receiving relay in a local circuit after a short signal, if the succeeding analogous signals cannot in general be received, or else their receipt is not essential.

Another possibility of reducing the number of relays and contacts in the network is, in some cases, through-transmission of the external signal along the actuating circuit /17/. This is possible when the signals received and transmitted by the network may be identical in certain periods of operation. In this case it is possible to connect the incoming wire with the network relay with outgoing contacts. An example of such a solution are the relay sets II/IVGI of the step-by-step automatic telephone station /122/, in which after selecting the necessary outgoing lead,

the talking wires are connected   and further control pulses pass
directly, without acting on the circuit of this group selector.

In those cases when the character of the external signal is
such that it can be received directly by an intermediate or actuating
element, it becomes possible to exclude the receiving element from
the network. Since we speak of relay networks, such an exclusion
becomes possible when the signal is transmitted over electrical
receiving circuits and when the values of the voltages and resis-
tances in these circuits are such that stable operation of the inter-
mediate or actuating elements is insured.

In this case the receiving circuit may be represented as a
closing or opening contact with a series-connected resistance (if
such is used in the external circuit). If the circuit is now
converted such that only one contact is used on the receiving relay,
then this relay can be excluded from the network, and its contact
can be represented as an external signal circuit.

By way of an example, Fig. 26 shows the diagram of a binary
pulse-number divider, in which the signal circuit is arbitrarily
denoted by u. The counting relays A and B receive the external
signals, and the windings of one relay should have an identical
number of ampere turns.

In some cases the   exclusion of the receiving relay may
lead to the   fact that during the process of
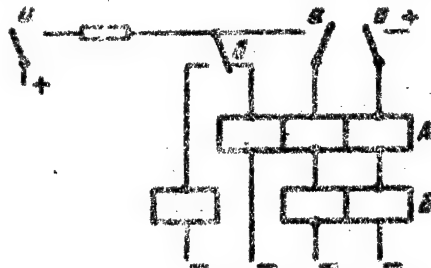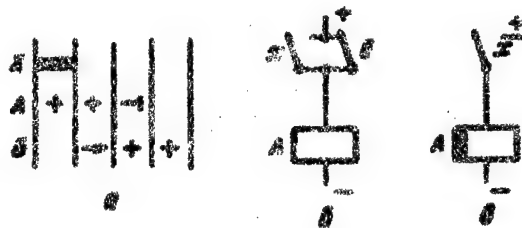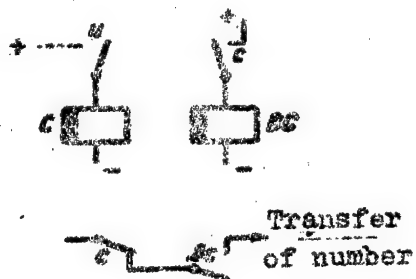operation of the network it feeds to the incoming circuit certain

Fig. 26

Fig. 27

Transfer
of number

Fig. 28

voltages, or   else individual input circuits may become inter-
connected. If this is not permissible in accordance with the opera-
ting conditions, the circuit can be separated, for example, by using
rectifiers.

9. USE OF TIME-DELAY RELAYS IN SEQUENTIAL NETWORKS

Relays in which the operation or drop-out are delayed are
used in relay networks primarily as receiving relays, responding to
prolonged pulses or to intervals between them, i.e., to receive a

temporal distinguishing feature. This functions is performed,
for example, by the series and holding relay in many automatic
telephone station networks, connected with the receipt of the number
from the subscriber.

Sometimes the time delay is used in intermediate relays so as
to reduce somewhat the number of contacts in the network.

The use of a   relay with a   time delay in drop-out makes
it possible to construct networks such that short-duration (within
a time shorter than the time delay) of breaks in the circuit do not
disturb the operation of the relay. This means that the circuit
of a relay with time delay exceeding the operating time or the
drop-out time of other relays in the network may be disconnected
during the unstable steps, produced by changes in states of the
latter. In other words, for circuits of time-delay relays, the

unstable steps can be considered as conditional ones, and in some cases this may make it possible to simplify the network considerably.

Thus, for example, to obtain sequential action x (Fig. 27a), the circuit of relay A should have in it contacts of relays X and B, as shown, for example, in Fig. 27b. If the relay A is made with time delay, then the same sequence will be retained without introducing into the circuit of relay A the contact of relay B (Fig. 27c). This principle serves as the basis for the construction of the network used to transmit a number from a counting network into a fixator in relay registers (Fig. 28). The number is transmitted after the drop-out of the series relay C (noting the end of the series) during the time when the auxiliary relay PC holds (because of xt the time delay) relay BC bringing the counting circuit back to the initial state after drop-out. Had time delay not been used here, the circuit of relay PC would have to pass through the contacts of the fixing relays, in order to have the relay BC drop out only after the fixing relays have operated.

In some cases the time delay of the relay can be used to increase the reliability of the operation of the network in transient conditions, when short-duration breaks or closures are possible in the circuit because of non-simultaneous reswitching of different contacts of certain relays during operation or drop-out.

10. CHANGE OVER FROM CONNECTION TABLES TO THE NETWORK

The connection table makes it possible to find directly the analytical expressions for the conditions for individual circuits, both actuating and those acting on the intermediate relays.

The corresponding expressions can be obtained as a sum of constituents for those states, in which the circuits should be closed (denoted in the connection table by the symbols $\longrightarrow$ and "plus" for intermediate relays and by heavy bar for the actuating relays).

From the table one can determine those constituents, which can be taken as conditional terms. The latter are the constituents of the unused states (which do not enter into the connection table), and also the constituents of states shown dotted for the actuating circuits.

Thus, one can obtain directly from the connection table the following general solutions for each circuit of the network in the form

$$f_x = \sum k_{l_i} + \sum \frac{k_{l_i}}{0}.$$

(36)

Thus, for example, from the table of connections of Fig. 20, the circuit f_b is written in the form

$$f_b = (10, 11, 15, 18, 19, 26, 27, 31, (2, 3, 16, 25, 29, 30, 4, 5, 6, 7, 12, 13, 14, 24, 28))_{B\,\mathrm{и}\,P.7.}$$

The transmission of these solutions and the change over to the choice of the structure will be considered in the following

chapters.

In some cases, particularly for actuating circuits, one can
see directly from the table on which relays the closed state of the
individual circuits depends, and the result can be written directly,
without any transformations whatever. Thus, for example, if in all
the steps during which the relay A operates the circuit f_χ should
be or can be closed, and in steps when this relay does not operate
it can or should be opened, we can write immediately

$$f_\chi = a.$$

If the preceding conditions pertain to steps in which the
relay A does not operate, we write respectively

$$f_\chi = \bar{a}.$$

Analogously, we can obtain also the circuits that depend on
two or more relays. Thus, for example, from the connection table of
Fig. 20 we can see that f_α should or can be closed in all cases,
when the relays L or P operate (or both simultaneously) and simul-
taneously we can write directly from the table

$$f_\alpha = \ell + p .$$

In cases when the number of steps in which the circuit should
be closed is large compared with the number of steps during which
it should be opened, it is more convenient to compile a "non-
operation circuit" as the sum of the constituents of those states,
in
during which the circuit should or can be opened, i.e.,

$$f\bar{x} = \sum k_{v_i} + \sum \frac{k_{u_j}}{0}, \qquad (3.17)$$

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where k_{γ_i} is the constituent of the forbidden state with number n_i.

Then
$$f_x = \overline{f\overline{x}}.$$

(3.18)

In analyzing the operation of any intermediate relay, one can obtain the following steps for this relay from the table:

Operation, in which conditions are produced for operation of the given relay (noted $\overset{d}{\longrightarrow}$).

Holding, in which the relay operates, ~~including~~ operates by being fed in local circuits ~~flank~~ $(+)$.

/release/

Drop-out, in which conditions are produced for drop-out of the relay $(\longrightarrow)$.

Conditions when the relay does not work.

In this case the circuit of a given relay should be closed during the operation and holding steps and opened during the steps of drop-out and non-operation. We note that steps that follow the operation steps differ from them only in the state of a given relay, and it follows from this that in the sum of the constituents of the operation step with the constituent of the next step, the contact of the considered relay is eliminated, i.e., we obtain a sub-constituent of the operation step. (We shall call sub-constituents /18/ of a circuit consisting of n relays those constituents of the network, obtained from a given network by excluding the contacts of this relay, whose operation is considered, i.e., a network of n - 1 relays.)

We introduce the concept of formulas for the operation, r_W, holding g_W, and drop-out h_W, which characterize the states of a network in different instants of operation of the relay W, and which do not include the contacts of this relay.

In accordance with this, it is most convenient to write down the formula for the operation of the relay W in the form

$$f_w = r_w + w g_w \qquad (3.19)$$

$$f_w = r_w + w \overline{h_w}. \qquad (3.20)$$

The first of these formulas insures the closing of the relay
during
circuit the operation steps and in all the holding steps, while the second insures the closing of the circuit during the operation and the opening (the inversion of the drop-out formula is taken) during the drop-out steps.

Since the contacts of the relay W, do not enter into the ex-
for
pressions r_W, g_W, and h_W, with respect to which the formulas are
given, they will consist of sub-constituents of the corresponding steps.

In accordance with the subdivision of the steps, we intro-
duce the following symbols:

r_j -- sub-constituents corresponding to the steps of operation of a given relay;

h_j -- sub-constituents corresponding to drop-out steps;

g_j -- sub-constituents corresponding to the remaining steps, inwhich a given relay operates, and

g'_j — those in which it does not operate.

Here j is the serial number of the corresponding sub-constituent.

The formula for the operation r_W can be represented as a sum of sub-constituents r_j, corresponding to all the steps of operation of this relay.

In addition to these sub-constituents, which are obligatory terms, one can add to the formula terms corresponding to certain unused states. It is clear that one cannot add to the operation formula the sub-constituents g'_j corresponding to those unused states, in which the relay W must not operate, since the addition of such a sub-constituent will cause operation of the relay during a step when it should not operate. If on the other hand the sub-constituents g'_j will be found among the unused states, it can be added to the operation formula. On the other hand, a sub-constituent g_j, corresponding to those unused states in which the relay W operates, cannot be added, since   a sub-constituent g'_j equal to it, corresponding to the non-working state of relay W, may be found among the unused states. Consequently, one can add to the operation formula only the sub-constituents of those unused states, during which the relay W does not operate.

The holding formula g_W is made up of the sum of the sub-constituents g_j, corresponding to the holding steps. By way of conditional terms one can take the sub-constituents r_j, corresponding to the operating steps, and also the sub-constituents

corresponding to the unused states during which the relay W operates.

If the relay has a drop-out time delay, then the conditional terms can be considered to be the sub-constituents of the unstable steps, holding and if it has an operating time delay, these sub-constituents correspond to the unstable steps, when the relay does not operate.

The drop-out formula h_w is made up as the sum of the sub-constituents h_j, corresponding to the drop-out steps, and the conditional sub-constituents are here those corresponding to the unused states, during which the relay W operates, as well as the sub-constituents r_j of the operating steps. For a relay with drop-out time delay, the conditional constituents can be those of the individual unstable steps.

Using the symbols introduced earlier, the general solution for the formulas for operation, holding, and drop-out become

$$r_w = \sum_i r_i + \sum_j \frac{g'_j}{0};$$ (3.21)

$$g_w = \sum_i g_i + \sum_j \frac{r_j}{0} + \sum_j \frac{g_j}{0};$$ (3.22)

$$h_w = \sum_i h_i + \sum_j \frac{r_j}{0} + \sum_j \frac{g_j}{0}.$$ (3.23)

where the indices i correspond to the used states, and the indices j to the unused ones, i.e., g_j is a sub-constituent corresponding to an unused state in which the relay W operates, while g'_j corresponds to one in which relay W does not operate.

Thus, for example, from the connection table of Fig. 20 we

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obtain for the relay A (relay A does not operates in the unused
states 4, 5, 6, and 7, and operates in states 12,13,14,24,and 28).

$$r_A = л\bar{p}\bar{y}\bar{6} + \frac{\bar{\bar{л}}\bar{p}y\bar{6}}{0} + \frac{\bar{л}p\bar{y}\bar{6}}{0} + \frac{\bar{л}py\bar{6}}{0} + \frac{лpy\bar{6}}{0};$$

$$g_A = лpy\bar{6} + лpy6 + лp\bar{y}\bar{6} + \bar{л}p\bar{y}\bar{6} + \bar{л}\bar{p}\bar{y}6 + \bar{л}\bar{p}y6 +$$

$$+ \frac{л\bar{p}y\bar{6}}{0} + \frac{\bar{л}\bar{p}y\bar{6}}{0} + \frac{лp\bar{y}\bar{6}}{0} + \frac{\bar{л}p\bar{y}\bar{6}}{0} + \frac{\bar{л}py\bar{6}}{0} + \frac{\bar{л}\bar{p}\bar{y}6}{0};$$

$$h_A = лp\bar{y}\bar{6} + \bar{л}\bar{p}\bar{y}6 + лpy\bar{6} + \bar{л}p\bar{y}\bar{6} + \bar{л}py6 +$$

$$+ \frac{л\bar{p}\bar{y}6}{0} + \frac{\bar{л}p\bar{y}\bar{6}}{0} + \frac{л\bar{p}y\bar{6}}{0} + \frac{\bar{л}p\bar{y}\bar{6}}{0} + \frac{\bar{л}py\bar{6}}{0} + \frac{\bar{л}\bar{p}y6}{0}.$$

It should be noted that   the elimination from the

holding formula of the sub-constituents r_j of the operating steps or

the introduction of these sub-constituents into the drop-out formula

may cause the circuit to be open for a short time (during the time

of movement of the contact).

When the conditional terms are taken into account with the

aid of the tables of neighboring constituents (see Chapter 5), there

is no need of paying attention to whether relay W does or doesnot

operate in the unused state, which is taken to be as the condition-

al term, and the operations can be carried out in their entirety

with constituents corresponding both to obligatory and to conditional

terms, excluding only from the results the contacts of the relay

whose circuit is being synthesized. This is possible because when

using the tables of neighboring constituents one can add to the

obligatory terms of the operation formula only those for which

the given relay does not operate, and one can add to the holding

and drop-out formula only those constituents in which it does operate.

If holding relays are used in the network (mechanical, magnetic, or electric) with two windings, the main one (operating) and bucking then for normal operation of such a relay it is essential that the circuit of the main winding W be closed only during the operating steps, and that the circuit of the bucking winding W be closed for a short period during the drop-out steps.

Thus, the structural formula of the network becomes in this case

$$F = r_W W + h_W \overleftrightarrow{W}.$$

(3.24)

For relays with electric holding, it is necessary to produce a holding circuit after operation through a closing (normally-open) contact of the relay, and the circuit energizing the bucking winding must also contain a closing contact of the same relay, so that the relay does not operate again through the bucking winding.

The structural formula of the network (Fig. 29) becomes in this case /16/:

$$F = (r_W + w) W + w h_W \overleftrightarrow{W}$$

(3.25)

or

$$F = (r_W + w)(W + h_W \overleftrightarrow{W}).$$

(3.26)

We note that in the expression h_W in formulas (3.24) ---- (3.26) one cannot introduce conditional sub-constituents r_i corres-
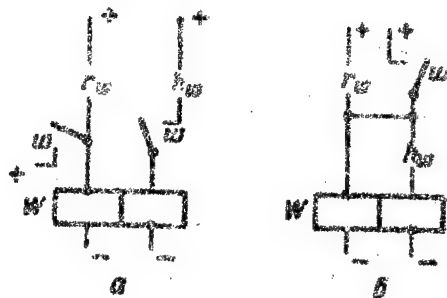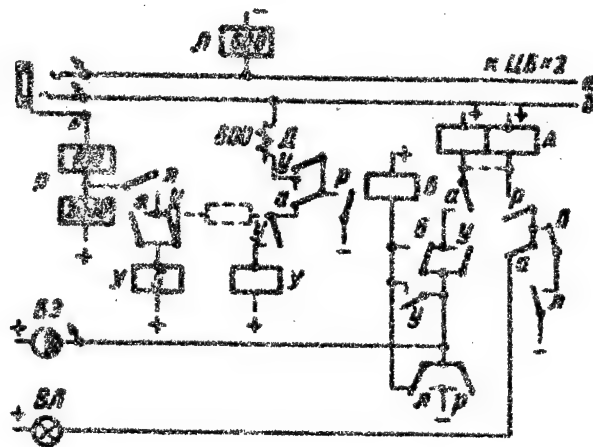
Fig. 29.

Fig. 30

ponding to the operating steps.

On the basis of the conditions obtained for the individual circuit, one can make up the diagram of the circuit, using, for example, the transition from the formula to the diagram after transformations for the purpose of reducing the number of contacts (see Chapter 4 and 5) or using the graphical method developed in Chapter 7. Experience has shown, however, that the greatest economy in contacts is obtained if there is a possibility of unifying several circuits and to construct the network in such a way, that one and the same contact acts on as many elements as possible. General considerations regarding these problems are given in Chapter 9.

The simplest method of joining circuits is to construct a normal network (Fig. 2a) in which individual contact circuits are joined into a (1, k)-pole network, the common point of which is on one of the poles of the battery. In this case the joining of the contacts is obtained by transforming the formulas with the symbol for these contacts taken outside the brackets. Further simplification of the contact circuit can sometimes be obtained by going over to a bridge circuit, for which can recommend the methods of M. A.Gavrilov /16/ or A.G. Lunts /30 -- 32/. The bridge contact network can also be obtained directly from the written down conditions by a graphic method (see Chapter 7) or by the method of F. Svoboda /59, 78/.

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By way of an example, Fig. 30 shows a RSL network constructed analytically /18/ in accordance with the connection table of Fig. 20.

In the case when the connection table is so compiled that only a small part of all the possible states is used and the number of unused states is large, the general method developed here calls for cumbersome manipulations. In this case one can recommend for each intermediate relay to construct partial tables of connections in which one introduces first only the elements which change their states directly ahead of the operating steps or the drop-out steps of a given relay, and if these tables are not realizable, one can make the network realizable by introducing other elements from among those contained in the overall table, as is recommended by, M. A. Gavrilov /69 73/. Other simplified methods of obtaining from the tables structural formulas for the intermediate relays in analogous cases are given by A. N. Yurasov /65/ and Ya. I. Mekler /135, 136/.

Chapter 4

ALGEBRAIC TRANSFORMATIONS OF CONTACT NETWORKS

1. BASIC MATHEMATICAL TOOLS

The basic mathematical tool of modern theory of contact net-
works is the formalism of algebraic logic (Boolean algebra), which
operates with two numbers -- zero and one.

In algebraic logic /5, 6, 123 , 124/ there are many functions
that characterize operations with elements that can assume two
values. The functions also assume two values. We give here the
characteristics of several basic functions of algebraic logic,
which are illustrated in Table 11, and also by the so-called Euler
circles, given in Fig. 31.

(Euler circles are used principally in set theory for a clear
representation of sets and operations on them. For the case of
contact networks, a rectangle represents the aggregate of all
the possible constituents, and the separated and shaded region
(Fig. 31a) represents the aggregate of those constituents, which
enter into the given function as obligatory. Two functions which
have no common constituents will be represented by two non-
intersecting domains, and those having common constituents will be

represented by intersecting domains, where the common domain corresponds to the constituents which are common to both sets. Such a representation permits a clear illustration of all the operations both of algebraic logic (Fig. 31), and of the algebra of contact networks. It will shown below that one can represent analogously also operations in the presence of conditional terms.

Table 11

a	b	ab	$a+b$	$a \to b$	$a \sim b$	$a \oplus b$	a/b	$\bar{a}$	$\bar{b}$
0	0	0	0	1	1	0	1	1	1
1	0	0	1	1	0	1	1	0	1
0	1	0	1	0	0	1	1	1	0
1	1	1	1	1	1	0	0	0	0

1. Logical addition (disjunction, joining) corresponds to the logical link word "or" (in the non-separating meaning). The function $a + b$ (sometimes denoted $a \cup b$ or $a \vee b$) is true when at least one of the terms is true (Fig. 31b).

2. Logical multiplication (conjunction, intersection, corresponds to the logical link word "and." The function $a \cdot b$ ($a \wedge b$, $a \wedge b$, $a \& b$) is true only when both factors are true (Fig. 31c).

3. Implication (inclusion) corresponds to the logical link word "if..., then ..." and "from ... it follows that ...." The function $a \to b$ ($a \supset b$) is false when a is true and b is false, and is true in all other cases (Fig. 31d).
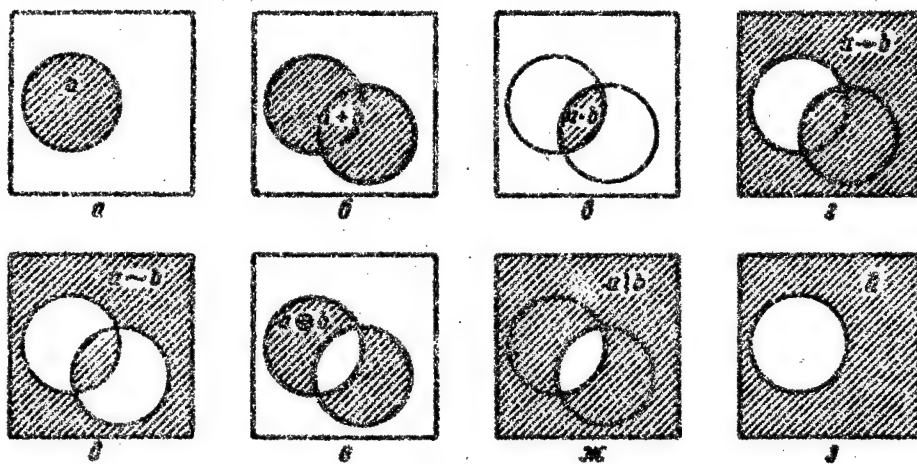
Fig. 31.

4. Equivalence (exclusion $\overset{\perp}{\text{"or"}}$). The function $a \sim b$ ($a \equiv b$, $a \overset{\cdot}{\top} b$) is true when a and b have equal values (Fig. 31e).

5. Alternative (addition modulo 2) corresponds to the logical link word "or" in the separating sense. The function $a \oplus b$ ($a \bigcirc b$, $a \overset{\cdot}{\perp} b$) is true only when a and b have different values (Fig. 31f).

6. The Sheffer function a/b, which is false only when a and b are simultaneously true, and true in all other cases (Fig. 31g).

7. Inversion, corresponding to logical negation. The function $\bar{a}$ (a', $\top a$, $1 - a$) is true when a is false, and is false when a is true (Fig. 31h).

From among all these operations, we have chosen for the algebra of contact networks the operations of multiplication, addition, and inversion, since, firstly, addition and multiplication /10, 86 -- 88/ are simulated by parallel and series joining of the contacts (circuit closed -- true -- one; circuit opened -- false -- zero), and the inversion corresponds to a replacement of a closing (normally opened) contact by an opening one (normally closed), and vice versa.

Secondly, they form a functionally complete system of functions, i.e., with them one can write down all the remaining functions:

$$a \rightarrow b = \bar{a} + b = \overline{\overline{a}\overline{b}}, \qquad (4.1)$$

$$a \sim b = ab + \bar{a}\bar{b}, \qquad (4.2)$$

$$a \oplus b = \overline{a \sim b} = a\bar{b} + \bar{a}b, \qquad (4.3)$$

$$a/b = \overline{ab} = \overline{a} + \overline{b}.$$

$$(4.4)$$

2. BASIC LAWS AND RELATIONS OF ALGEBRA OF CONTACT NETWORKS

In the algebra of contact networks the following laws hold:

1) Commutative (rearrangement)

$$\left. \begin{array}{l} xy = yx, \\ x + y = y + x; \end{array} \right\}$$

$$(4.5)$$

2) Associative

$$\left. \begin{array}{l} (xy)z = x(yz), \\ (x + y) + z = x + (y + z); \end{array} \right\}$$

$$(4.6)$$

3) Distributive

$$(x + y)z = xz + yz$$

$$(4.7a)$$

(the distributive law of multiplication relative to addition)

$$xy + z = (x + z)(y + z)$$

$$(4.7b)$$

(distributive law of addition relative to multiplication).

4) Repetition

$$\left. \begin{array}{l} xx...x = x, \\ x + x + ... + x = x; \end{array} \right\}$$

$$(4.8)$$

5) Inversion

$$\overline{xy} = \overline{x} + \overline{y},$$

$$(4.9a)$$

$$\overline{x + y} = \overline{x} \cdot \overline{y},$$

$$(4.9b)$$

Laws (4.5) -- (4.8) make it possible to effect identical (in admittance) transformations of contact networks, while law (4.9) enables us to find networks which are reciprocal in structural

admittance ($\overline{xy}$ is reciprocal to $\overset{+}{xy}$).

Any expressions produced by the foreoing operations, as well as the individual symbols, can assume only two values, which corresponds to the fact that the contact circuit can be either opened or closed. The equal sign in all these formulas denote that for each state of the individual elements of the network, the circuits corresponding to both parts of the equation will simultaneously be either closed or opened.

In other words, the equal sign indicates that the circuits have an identical structural admittance, i.e., when the states of the elements change they will be simultaneously closed or simultaneously opened.

In accordance with this, we can write down the following equivalences:

$$0 \cdot x = 0; \qquad\qquad (4.10a)$$

$$1 \cdot x = x; \qquad\qquad (4.10b)$$

$$0 + x = x; \qquad\qquad (4.10c)$$

$$1 + x = 1; \qquad\qquad (4.10d)$$

$$x \cdot \overset{=}{x} = 0; \qquad\qquad (4.11a)$$

$$x + \vec{x} = \overline{1} \qquad\qquad (4.11b)$$

On the basis of the foregoing laws we can obtain also many other equivalences, which may be useful in network transformation.

$$a + ax = \overset{=}{a}; \qquad\qquad (4.12a)$$

$$a(a + x) = a; \qquad\qquad (4.12b)$$

$$a + \bar{a}x = a + x; \qquad\qquad (4.13a)$$

$$a(\bar{a} + x) = ax; \qquad\qquad (4.13b)$$

$$ax + \bar{a}y + xy = ax + \bar{a}y; \qquad\qquad (4.14a)$$

$$(a + x)(\bar{a} + y)(x + y) = (a + x)(\bar{a} + y). \qquad\qquad (4.14b)$$

Finally, we can note the following expansion formulas,

$$f(a, b, \ldots, n) = af(1, b, \ldots, n) + \bar{a}f(0, b, \ldots, n). \qquad (4.15a)$$

$$f(a, b, \ldots, n) = [a + f(0, b, \ldots, n)][\bar{a} + f(1, b, \ldots, n)]. \qquad (4.15b)$$

or, in more general form

$$f(a, b, \ldots, n) = ab \ldots n \cdot f(1, 1, \ldots, 1) + \bar{a}b \ldots nf(0, 1, \ldots, 1) +$$
$$+ a\bar{b} \ldots n \cdot f(1, 0, \ldots, 1) + \bar{a}\bar{b} \ldots nf(0, 0, \ldots, 1) + \ldots +$$
$$+ \overline{a\bar{b}} \ldots nf(0, 0, \ldots, 1) + \bar{a}\bar{b} \ldots \bar{n} \cdot f(0, 0, \ldots, 0). \qquad (4.16a)$$

$$f(a, b, \ldots, n) = [a + b + \ldots + n + f(0, 0, \ldots, 0)][\bar{a} + b + \ldots + n +$$
$$+ f(1, 0, \ldots, 0)] \ldots [\bar{a} + \bar{b} + \ldots + n + f(1, 1, \ldots, 0)][\bar{a} + \bar{b} + \ldots +$$
$$+ \bar{n} + f(1, 1, \ldots, 1)]. \qquad (4.16b)$$

3. Concept of Constituents

As already mentioned, in a circuit consisting of n relays one can produce 2^n different states. To each of these states corresponds a circuit of series-connected contacts of all n relays, such that will be closed only in a given state. The analytical expression corresponding to such a circuit is called the constituent of the expansion of unity, or simply constituent.

We shall denote the individual constituents by the letter k numbered with the number of the same state, in which the corres-

$$k_0 = \overline{\overline{a6s}},$$

$$k_1 = \overline{a\overline{6s}},$$

$$k_5 = \overline{a6a}, \quad etc.$$

It can be shown /16, p 62/ that a formula for any contact network can be represented as a sum of a certain number of constituents, and the network itself can be presented in the form of a series of parallel circuits, each of which consist of only series-connected contacts of all the relays of the network, i.e.,

$$f = \sum k_i \, (i = \eta_1, \eta_2, \ldots, \eta_r). \tag{4.17}$$

This follows, in particular, from formula (4.16a), and also from the fact that each circuit can be characterized by those states, in which it should be closed (see Chapter 2).

The sum of all 2^n constituents is unity, i.e., it corresponds to a continuously closed circuit

$$\sum_{i=0}^{2^n-1} k_i = 1. \tag{4.18}$$

This explains the name "constituents for the expansion of unity."

The constituents have the following properties, that

$$k_i \cdot k_j = 0 \quad (for \ i \neq j). \tag{4.19}$$

Inversion of the constituent k_j, which we call an anti-constituent (it is called the constituent for the expansion of zero) corresponds to a circuit made of the contacts of all relays

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in parallel, open in the state numbered i.

One can show that a formula for any contact circuit can be represented as the product of a certain number of anti-constituents, and the network itself can be represented in the form of a series connection of circuits, each of which consist of only parallel-connected contacts of all the relays of the network, i.e.,

$$l = \sum_{i = \eta_0, \eta_1, \ldots, \eta_r} k_i = \prod_{i = \eta_{r+1}, \ldots, \eta_{2^n-1}} \overline{k}_i. \tag{4.20}$$

The anti-constituents have the following properties:

$$\prod_{i=0}^{2^n-1} \overline{k}_i = 0, \tag{4.21}$$

$$\overline{k}_i + \overline{k}_j = 1 \quad \text{for} \quad i \neq j. \tag{4.22}$$

4. CONCEPT OF INCLUSION AND ITS NETWORK INTERPRETATION

Let us consider a concept of algebraic logic — inclusion, $a \subset b$ /5/ — which denotes that "each a is b" or that "b includes a" (Fig. 32). This means that the numerical value of the expression will never be greater than the expression for b (they can be equal). Thus, for example, if we compared the expressions ab and a + b, then as can be seen from Table 11, for all values of a and b the value ab will be either less than or equal to a + b, the value of i.e., $ab \subset (a + b)$.

Fig. 32.
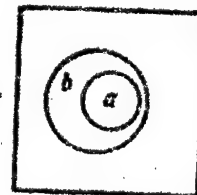
From the same table we can see, for example, that

$$ab \subset a; \quad ab \subset b; \quad a \subset (a + b) \text{ and } b \subset (a + b).$$

In other words, the inclusion $a \subset b$ denotes that "a is not greater than b" or a is less than or equal to b." Therefore we shall use for inclusion not the symbol $\subset$ but the symbol $\leqslant$ or its inverse $\geqslant$, i.e., if

$$a \leqslant b,$$

then

$$b \geqslant a.$$

Going over to contact networks, we can say that if formulas of two circuits f_1 and f_2 are connected by the relation

$$f_1 > f_2$$

this means that for all possible states of the contacts contained in these circuits, they will either be simultaneously closed or simultaneously opened $(f_1 = f_2)$ or else f_1 will be closed when f_2 is open $(f_1 \qquad f_2)$, i.e., there cannot be a state when the circuit f_1 is opened $(f_1 = 0)$ and f_2 is closed $(f_2 = 1)$.

In other words, the relation $f_1 \qquad f_2$ denotes that the function f_1 contains all the constituents making up the function f_2, and a certain number of additional ones.

Let us agree to say for short that the circuit f_1 is greater than the circuit f_2 or that f_2 is smaller than f_1.

The relations greater and smaller have the following properties /5/:

1. If $a \geqslant b$, then

$$ab = b,\qquad (4.23a)$$

$$a + b = a,\qquad (4.23b)$$

$$ab = 0,\qquad (4.23c)$$

$$a + b = 1.\qquad (4.23d)$$

2. Transitivity

If $a \geqslant b$ and $b \geqslant c$, then $a \geqslant c$. $\qquad (4.24)$

3. If $a \geqslant b$, then $a \leqslant \overline{b}$.

If $a \leqslant b$, then $\overline{a} \geqslant \overline{b}$. $\qquad (4.25)$

4. When $a \neq b$:

If $a \leqslant t$ or $b \leqslant t$, then $ab \leqslant t$; $\qquad (4.26a)$

If $a \geqslant t$ or $b \geqslant t$, then $(a + b) \geqslant t$; $\qquad (4.26b)$

If $ab \geqslant t$ then $a \geqslant t$ and $b \geqslant t$; $\qquad (4.26c)$

if $(a + b) \leqslant t$, then $a \leqslant t$ and $b \leqslant t$. $\qquad (4.26d)$

In contact networks the properties (42.3a. and b) signify that one can connect in parallel with each contact circuit a smaller circuit, or one can connect in series with it a larger circuit, without changing the overall structural admittance.

We note that the concept of the circuit f*, smaller than a given circuit f, is ambiguous, and pertains to the theoretically in-finite set of circuits, satisfying the inequality

$$f \geqslant f^* \geqslant 0.\qquad (4.27)$$

The same can be said also relative to the larger circuit f**, the limit of which is a short circuit. All circuits f**, greater

than the circuit f, satisfy the inequality

$$1 \geqslant f^{**} \geqslant f.$$ (4.28)

For inequalities of the type

$$f_1 \leqslant x \leqslant f_2$$ (4.29)

L. Kyutura /5/ gives the solution

$$x = f_1 \omega + f_2 \overline{\omega},$$ (4.30)

where ω is any expression (any contact circuit), called indeterminate.

For the inequality points, Kyutura gives a solution given by P.S. Poretskiy

$$x = f_1 + f_2 w'.$$ (4.31)

Formulas (4.30) and (4.31) are essentially identical, if it is considered that when f_1 f_2 we obtain $f_2 = f_1 + f_2$.

These formulas make it possible, in particular, to   determine any greater circuit f^{**} relative to f, by putting $f_1 = f$ and $f_2 = 1$, i.e.,

$$f^{**} = f\omega + \overline{\omega} = f + \omega'.$$ (4.32)

Analogously, we can obtain also a "smaller" circuit f^*, by putting $f_1 = 0$ and $f_2 = f$, i.e.,

$$f^* = f\omega$$ (4.33)

Several procedures for obtaining greater and smaller circuits are given in the author's paper /53/. In particular, a greater circuit can be obtained by replacing any of the symbols in the formula (any element regardless of whether it is direct or inverse, and regardless of the presence of a repetition of this element in

the formula) by unity, and a smaller circuit is obtained by replacement with zero.

Thus, for example:

$$ad + \bar{a}(c + \bar{b}) < ad + c + \bar{b} < a + c + \bar{b} < 1;$$

$$ad + \bar{a}(c + \bar{b}) > ad + \bar{a}c > ad > 0.$$

We shall agree to denote a circuit greater than the circuit f by the symbol $\dfrac{f}{1}$, and one smaller than f by the symbol $\dfrac{f}{0}$.

More will be said about these symbols in Chapter 5, Section 3.

5. Structural Transformations of Contact Networks

Since in the synthesis of networks one obtains individual circuits in the form of two-pole networks, we shall first consider equivalent structural transformations of these circuits, i.e., transformations under which the structure of a circuit changes, but the structural admittance remains the same.

In the synthesis of relay networks, the transformation of contact circuits has as its principal purpose the creation of the simplest network. In this connection, we can formulate the following problems, which are solved in the structural transformation of contact networks:

Finding the structure with the smallest number of contacts;

exclusion of excessive (duplicating) circuits and contacts from the network;

redistribution of the contacts среди among individual relays;

joining of a closing and opening contact of one relay in a double-throw contact group to reduce the number of springs in the network;

reduction of individual circuits to such a form, that when they are joined the overall network contains a minimum of contacts;

production of a structure more convenient for wiring of the network, particularly with a minimum number of wires between networks or parts of a single network.

In class π networks, the structural transformations can be carried out by using notations in the form of structural formulas, and the number of symbols for contacts entering into the formula will exactly be equal to the number of the contacts.

To go over to bridge circuits, it is necessary to have additional transformations, such as matrix transformations. In class H networks, a decrease in the number of contacts can be accomplished in some cases by introducing valve elements, as will be shown below.

6. STRUCTURAL TRANSFORMATIONS OF CLASS π CONTACT TWO-TERMINAL NETWORKS

Let us consider now individual structural transformations of class π contact two-terminal networks, aimed at reducing the number of contacts in the network, and the laws of algebraic contact

networks, which are employed therein.

The basic procedure for reducing the number of contacts in a circuit is to exclude excessive contacts or circuits from the network or to join into a common circuit the contacts contained in several circuits.

Laws (4.5) and (4.6) show the possible rearrangements of contacts within a circuit. The elimination of excessive contacts in and circuits is on the basis of the equivalences (4.10) -- (4.14).

The joining of contacts is accomplished by using the distributive laws (4.7), either by taking outside of the bracket (4.7a) or by separating into an individual component (4.7b) those terms, which are common to several circuits, connected in parallel or in series.

In some cases, however, it is advantageous to expand individual circuits, so that they can be unified.

Thus, for example, if there exists a circuit

$$f = abc + (\overline{a} + b)\, d,$$

then the first term is best expanded, and written in the form $a(\overline{a} + b)c$, and then

$$f = (\overline{a} + b)\,(ac + d).$$

which leads to the elimination of one contact.

In the general case the "expansion" of the contact circuit, i.e., the introduction in it of additional contacts, is based on the equivalences (4.23a) and (4.23b) by connecting a larger circuit in series or a smaller one in parallel.

7. Transformation of Bridge Circuits

In bridge contact networks, in addition to the transformations that follow from the transformation of class π contact two-terminal networks, which can be applied also to individual branches of the circuit, there is still another possible form of transformations, connected with introducing bridge elements into the network. These transformations, without changing the structural admittances between poles, lead in many cases to a change in the number of nodes of the circuit and of the circuits between the individual nodes.

The simplest case of such a transformation is the $Y-\Delta$ transformation and vice versa (Fig. 33), which can be written in terms of the direct-admittance matrices:

$$
\begin{bmatrix}
1 & 0 & 0 & x \\
0 & 1 & 0 & y \\
0 & 0 & 1 & z \\
x & y & z & 1
\end{bmatrix}
=
\begin{bmatrix}
1 & xy & xz \\
xy & 1 & yz \\
xz & yz & 1
\end{bmatrix}
\tag{4.34}
$$

A general method of introducing new nodes in bridge networks was given by A. G. Lunts /30 — 32/. This method is very effective, since it makes it possible to construct bridge networks and to introduce rectifier elements in them.

A shortcoming of this method is that the complexity of the network depends on the sequence with which the new nodes are introduced, and the constructed network may turn out to be even more complicated than a class π network. At the present time there is still no algorithm that leads to a simplification of the network

upon introduction of bridge elements. V. G. Lazarev /93/ has shown that if when using the method of A. G. Lunts one first finds the initial and final elements, for example, by the method of M. A. Gavrilov /16/, a certain sequence in construction can be obtained, leading to simpler networks.

The general rules indicating the possibility of unifying identical circuits contained in different branches of the network have been recently given by M. A.Gavrilov /69, 77/. The procedure proposed by him makes it possible to evaluate methods of constructing bridge contact networks.

8. INVERSION

By inversion in the algebra of contact networks is meant the finding of a contact network ϕ, inverse in its admittance to the initial circuit f, i.e., the circuit ϕ should be closed in those states of the network, during which the circuit f is opened, and vice versa.

For networks of class Π, starting out with the equivalences (4.9), one obtains the following rule for inversion: to invert a class Π contact network all the parallel connections are replaced by series ones, and all the series ones are connected by parallel ones, and at the same time closing contacts are replaced by opening ones, and vice versa.

If rectifiers are contained in the network, they should be

reversed 180°.

To invert planar networks of class H, one can recommend a graphical method /16, p 82/ consisting of replacing each closed loop by a node, and each node by a closed loop, simultaneously replacing each two-pole network comprising the branches   by   its inverse (Fig. 34). If the   network contains rectifiers, in the graphic inversion their polarity is chosen in accordance with the following rule: when the input pole of the network is moved counterclockwise, the rectifier should be turned clockwise, and vice versa (Fig. 35).

It is obvious that in both cases of inversion the number of elements in the network,   and also the number of contacts in individual relays, remains unchanged.

For non-planar networks of class H, there are no methods for direct inversion and one can recommend in such cases to write down the network in the form of structural formula, obtaining the inverse expression, and constructing the inverse network from this inverse formula.   In this case the number of elements in the network may change.
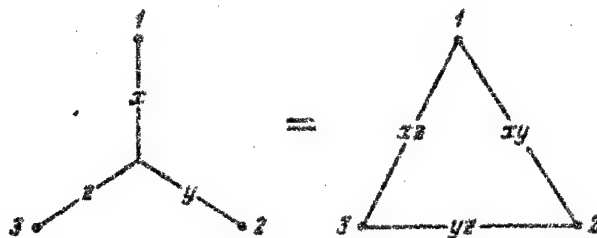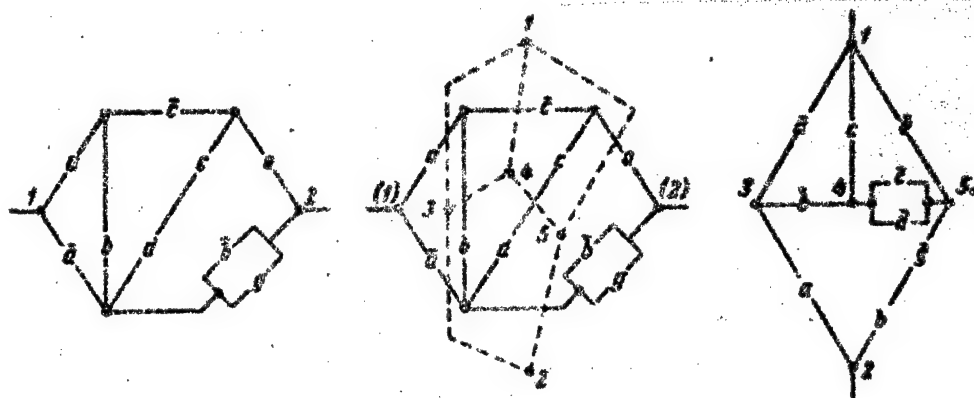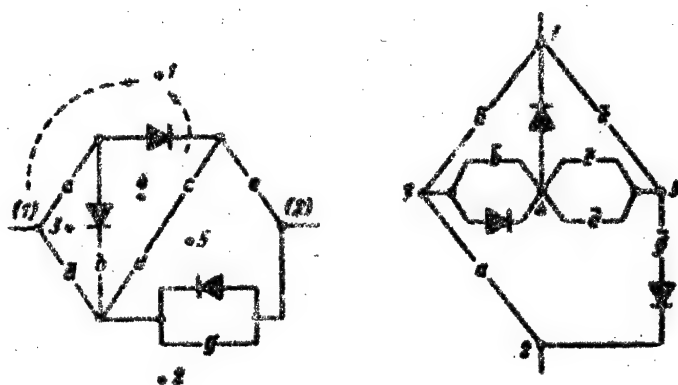

Fig. 33.

Fig. 34.

Fig. 35.

Chapter 5

GENERAL SOLUTIONS AND THEIR TRANSFORMATIONS

1. Concept of General Solution

As already indicated (see Chapter 2, Section 3), the operation of individual circuits is written down in general as the sum of obligatory and conditional terms, corresponding to those states, in which a given circuit should or could be closed. To write down the conditional terms we made use of the symbol $\frac{k_\mu}{0}$, indicating that the constituent numbered μ can be replaced by zero. The presence of conditional terms in the expression, several different circuits may correspond to this expression, depending on which of the conditional terms are taken completely, and which are replaced by zeroes.

A circuit (or its analytical expression) satisfying given conditions will be called a solution. Because of the presence of conditional terms, several solutions may be obtained for one set of conditions.

Thus, for example, if the circuit of the relay X contains one obligatory term abc and two conditional terms $\overline{a}bc$ and $ab\overline{c}$, then the solution can be written in the form abc   (not one of the conditional terms has been added)   ab, ac, or a(b + c)   (one of

the conditional terms or both together have been added)

In this case, <s>any</s> none of these solutions can be transformed into another by usual transformations of the theory of relay-contact networks (see Chapter 4), since these solutions have been obtained with allowance for the operating conditions of the network.

It should be noted that if the addition of certain conditional terms simplifies the expression, the addition of others may complicate it.

The choice of the particular solution is best made only by comparison with other circuits in the synthesis of the network as a whole, so as to obtain the greatest unification of the individual elements.

Thus, for example, if in addition to the circuit of the relay X, there exists a relay Y, <s>the</s> whose operating formula $f_Y = (b + c)d$, then the circuit will be simplest, if we take the last of the solutions for the circuit of relay X:

$$F = (6 + e)(aX + eY).$$

It is also advantageous to know all possible solutions in the compilation of bridge circuits.

However, <s>if</s> while individual solutions cannot be reduced into one another, <s>and</s> each of these can be obtained from the solution written in the following form

$$f_x = a6c + \frac{\overline{a6}\,e}{0} + \frac{a\overline{6e}}{0}. \tag{5.1}$$

An expression of the form

$$f_X = \sum_{i=1}^{r} k_{q_i} + \sum_{j=1}^{s} \frac{k_{u_j}}{0}$$

(5.2)

will be called the general solution, since all the particular solutions can be obtained from it by choosing different conditional terms.

2. Determination of the Particular Solutions

Using the concept of smaller and larger circuits (see Chapter 4, Section 4) we can say that if not a single conditional term is taken in expression (5.2), we obtain the minimal value of expression f_X, i.e.,

$$f_{X_{min}} = \sum_{i=1}^{r} k_{q_i}.$$

(5.3)

The addition of all the conditional terms gives a maximal value for this expression, i.e.,

$$f_{X_{max}} = \sum_{i=1}^{r} k_{q_i} + \sum_{j=1}^{s} k_{u_j}.$$

(5.4)

Consequently, any particular solution f^*_X, satisfying formula (5.2), should also satisfy the inequality

$$f_{X_{min}} \leqslant f^*_X \leqslant f_{X_{max}}.$$

(5.5)

In accordance with (4.31), the general solution can be written in the form

$$f^*_X = f_{X_{min}} + f_{X_{max}} \omega.$$

(5.6)

Since no limitations have been imposed on ω, we can see

from formula (5.6) that the number of different particular solutions can be infinite, considering that arbitrary variables can be introduced into   .

In practice, however, it is primarily important to obtain particular solutions consisting of the same variables as in the function $f_{X_{max}}$ and $f_{X_{min}}$. Such particular solutions will be called fundamental. It can be shown that if there are s conditional constituents in formula (5.2), the number of different fundamental solutions will be 2^s.

Individual particular solutions have that property, that both the product and the sum of several particular solutions is a particular solution.

In fact, if there are particular solutions

$$f_X^1 = f_{min} + \omega_1 f_{max}$$

and

$$f_X^2 = f_{min} + \omega_2 f_{max},$$

then
$$f_X^1 + f_X^2 = f_{min} + (\omega_1 + \omega_2) f_{max} = f_{min} + \omega_3 f_{max} = f_X^3; \tag{5.7a}$$
$$f_X^1 f_X^2 = f_{min} + \omega_1 \omega_2 f_{max} = f_{min} + \omega_4 f_{max} = f_X^4. \tag{5.7b}$$

Another particular solution will be
$$f_X^3 = f_{X^\omega}^1 + f_{X^\omega}^{2-}, \tag{5.7c}$$

which can be readily verified by substituting the values for f_X^1 and f_X^2 into (5.7c).

Now that we have explained the nature and the basic properties

of the general and particular solutions, let us proceed to the question of abbreviating the notation for the general solution, since a solution written in the form of a sum of constituents is inconvenient in operation and does not make it possible in practice to change over to the network.

3. EQUIVALENCES

In writing down the conditional terms in the general solution, we have used the symbol $\dfrac{a}{0}$, denoting that one can take either the expression a or zero.

Let us broaden this concept now for a more general case, when it is necessary to write down non-unique solutions.

We have seen that the presence of conditional states may cause several different circuits to satisfy the same set of conditions, these circuits having identical values for these conditions. We shall use the symbol already known to us, the fraction bar, to write down such circuits. We shall treat this symbol as a sign indicating that the expressions on both sides are equivalent for given conditions, and either of these can be taken. Such expressions with the bar will be called equivalences.

(An equivalence is one of the methods of writing down multiple-valued solutions, analogous to the use of integration constants in the solution of differential equations. As in the latter case, the choice of the particular solution is based on the supple-

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mentary solutions (for example, obtaining a network with a minimum number of contacts).)

Let us examine the properties of equivalences.

We note first of all that it is immaterial on which side of the symbol (bar) a particular expression stands, i.e., the commutation law holds for equivalences:

$$\frac{a}{b} = \frac{b}{a}. \tag{5.8}$$

Considering all the possible combinations of the four constituents of two variables a and b ($\overline{ab}$, $\overset{\smile}{ab}$, $\widehat{ab}$, and $\overline{ab}$), we can verify that corresponding to the condition a/b is the case when one adds to the obligatory constituents ab the conditional constituents $\overset{\smile}{ab}$ and $\overline{ab}$, i.e., the equivalence can be represented in the form of a general solution

$$\frac{a}{b} = ab + \frac{\overline{ab}}{0} + \frac{\overline{ab}}{0}. \tag{5.9}$$

Starting with the statements made in the preceding section concerning the general solution, we can write

$$ab \leqslant \frac{a}{b} \leqslant ab + a\overline{b} + \overline{a}b$$

or

$$ab \leqslant \frac{a}{b} \leqslant (a+b), \tag{5.10}$$

hence, in accordance with (4.30) and (4.31)

$$\frac{a}{b} = ab + (a+b)\,\omega = ab\overline{\omega} + (a+b)\,\omega. \tag{5.11}$$

where ω, as before, is an arbitrary expression.

Thus, equivalence is none other than an abbreviated notation

for a group of Boolean functions, satisfying inequality (5.10) and determined by expression (5.11).

The particular solutions are obtained by assigning to the symbol ω in (5.11) some definite value.

Since the expression obtained after substitution of ω no longer reflects the specific features of the operating conditions of the network, the inverse transition from the particular solution to the general solution is impossible. Therefore, to go over from the general solution to the particular solution we shall use not the equality sign, but an arrow, indicating the one-sidedness of the transformation.

For the particular values x we obtain from formula (5.11):

$$
\begin{aligned}
\text{при } \omega &= 1; & \frac{a}{b} &\to a + b, \\
\text{при } \omega &= 0; & \frac{a}{b} &\to ab, \\
\text{при } \omega &= a; & \frac{a}{b} &\to a, \\
\text{при } \omega &= b; & \frac{a}{b} &\to b.
\end{aligned}
\tag{5.12}
$$

It is easy to verify that these four solutions are fundamental.

With other substitutions we can obtain other particular solutions. Let us note some of them:

$$
\begin{aligned}
\frac{a}{b} &\to a + bx, \\
\frac{a}{b} &\to b + ax, \\
\frac{a}{b} &\to a(b + x), \\
\frac{a}{b} &\to b(a + x).
\end{aligned}
\tag{5.13}
$$

The case when in one of the parts of the equivalents there is

a zero, corresponds to the inequality

$$0 < \frac{a}{0} \leqslant a \qquad (5.14)$$

or, from (5.11)

$$\frac{a}{0} = a\omega. \qquad (5.15)$$

For the case a/1 we shall have respectively

$$a \leqslant \frac{a}{1} \leqslant 1 \qquad (5.16)$$

or

$$\frac{a}{1} = a + \omega. \qquad (5.17)$$

Analogously, $\dfrac{\frac{a}{b}}{c}$ denotes that

$$abc \leqslant \frac{\frac{a}{b}}{c} \leqslant a + b + c \qquad (5.18)$$

or

$$\frac{\frac{a}{b}}{c} = abc + (a + b + c)\omega. \qquad (5.19)$$

This result can be derived from formula (5.11):

$$\frac{\frac{a}{b}}{c} = \frac{ab + (a + b)\omega}{c} = [ab + (a + b)\omega]c + [ab + (a + b)\omega + c]\omega_1.$$

Putting $\omega = \omega_1$, we get

$$\frac{\frac{a}{b}}{c} = abc + (a + b)\omega c + ab\omega + (a + b)\omega + c\omega = abc + (a + b + c)\omega.$$

It is easy to verify that $(a + b + c)$ contains, in addition to the constituent abc, six additional constituents, i.e., the expression $\dfrac{\frac{a}{b}}{c}$ corresponds to $2^6 = 64$ different fundamental solutions.

The symbol of equivalents may be found convenient for an

abbreviated notation in the absence of conditional terms and in cases similar to the following one. If there is an expression

$$f = ab + \bar{a}c + bc + a\bar{c},$$

then, since $ab + \bar{a}c + bc = ab + \bar{a}c$ or $ab + \bar{a}c + a\bar{c} = bc + a\bar{c}$, we obtain two equivalent solutions

$$f = ab + \bar{a}c + a\bar{c} = \bar{a}c + bc + a\bar{c}.$$

Using the equivalence sign, we can write

$$f = ab + \bar{a}c + bc + a\bar{c} = a\bar{c} + \bar{a}c + b\frac{a}{c}.$$

4. TRANSFORMATION OF EQUIVALENCES

When operating on equivalences we should consider each of their parts separately. With this, all the laws and formulas of the algebra of contact networks apply to equivalences. One can readily verify the correctness of the following equivalences:

$$a + \frac{b}{c} = \frac{b}{c} + a = \frac{a+b}{a+c}; \qquad (5.20)$$

$$a\frac{b}{c} = \frac{b}{c} \cdot a = \frac{ab}{ac}. \qquad (5.21)$$

From this it follows, in particular, that

$$
\left.
\begin{array}{ll}
a \cdot \dfrac{b}{1} = \dfrac{ab}{a}; & a + \dfrac{b}{1} = \dfrac{a+b}{1}; \\[2mm]
a \cdot \dfrac{b}{0} = \dfrac{ab}{0}; & a + \dfrac{b}{0} = \dfrac{a+b}{a}; \\[2mm]
a + \dfrac{\bar{a}}{b} = \dfrac{1}{a+b}; & a \cdot \dfrac{\bar{a}}{b} = \dfrac{0}{ab}; \\[2mm]
a + \dfrac{\bar{a}}{0} = \dfrac{1}{a}; & a \cdot \dfrac{\bar{a}}{1} = \dfrac{0}{a}; \\[2mm]
a + \dfrac{\bar{a}}{1} = \dfrac{1}{1} = 1; & a \cdot \dfrac{\bar{a}}{0} = 0, \text{ и т.п.}
\end{array}
\right\}
$$

$$(5.22)$$

The inversion of equivalences is also carried out in accor-

dance with the usual rules, with retention of the equivalence symbol

$$\overline{\left(\frac{a}{b}\right)} = \frac{\overline{a}}{\overline{b}}; \qquad \overline{\left(\frac{a}{0}\right)} = \frac{\overline{a}}{1}; \qquad \overline{\left(\frac{a}{1}\right)} = \frac{\overline{a}}{0};$$
$$\overline{a + \frac{b}{c}} = \overline{a} \frac{\overline{b}}{\overline{c}}; \qquad \overline{a \frac{b}{c}} = \overline{a} + \frac{\overline{b}}{\overline{c}}.$$

(5.23)

Multiplication of two equivalences corresponds to separate multiplication of each of the parts of the first equivalence by the second

$$\frac{a}{b} \frac{c}{d} = \frac{a\frac{c}{d}}{b\frac{c}{d}} = \frac{c\frac{a}{b}}{d\frac{a}{b}} = \frac{\frac{ca}{cb}}{\frac{da}{db}}.$$

(5.24)

Hence, for example,

$$\frac{a}{b} \frac{a+c}{b+d} = \frac{a\frac{a+c}{b+d}}{b\frac{a+c}{b+d}} = \frac{a\frac{1}{b+d}}{b\frac{a+c}{d}} = \frac{\frac{a}{a(\overline{b}+d)}}{\frac{b(a+c)}{bd}}.$$

The same pertains to addition

(5.25)

$$\frac{a}{b} + \frac{c}{d} = \frac{a+\frac{c}{d}}{b+\frac{c}{d}} = \frac{c+\frac{a}{b}}{d+\frac{a}{b}} = \frac{\frac{c+a}{c+b}}{\frac{d+a}{d+b}},$$

hence

$$\frac{a}{b} + \frac{c}{1} = \frac{a+\frac{c}{1}}{b+\frac{c}{1}} = \frac{\frac{a+c}{1}}{\frac{b+c}{1}} = \frac{c+\frac{a}{b}}{1} = \frac{\frac{c+a}{c+b}}{1}.$$

5. Simplification of the Notation for the General Solution

The general solution, written in expanded form, should be simplified in order to retain in the formula minimum number of symbols for contacts. This is attained by taking outside the brackets the common factors and adding those constituents, the

summation of which leads to the elimination of separate symbols.

(The process of reducing a Boolean function to normal form with the least number of letters is called minimization /125 — 127/. In the present paper we shall aim not at finding the possible minimal form, but at a convenience of notation for subsequent change over to the network.)

We note that simplification of expressions, considered in this section, has as its purpose to simplify the notation, and not to simplify the synthesized network, since it is known that in the general case the formula with the least number of elements does not always correspond to the most economical network. Only in a particular case, for class II two-terminal networks, does the minimum of elements in a formula determine the minimum of elements in a network.

We shall therefore consider here those simplifications, which permit merely an abbreviation of the notation.

It is easy to see first of all that a simplification of any structural formula is obtained by adding to the constituents contained in a given formula a constituent which differs by the inversion of one element. Then this element is excluded and the number of elements in the final expression is reduced.

Constituents consisting of the same elements and differing only in the state of one of these elements are called neighboring /66/. In accordance with these, when adding two neighboring constituents one excludes the element in whose state they differ.

Analogously, one excludes from the expression two elements when four constituents are added, differing in a combination of states of these two elements, and three elements are excluded by adding eight constituents, which differ in combinations of the states of these three elements, etc.

Therefore, when carrying out transformations for the purpose of simplifying the notation, it becomes necessary to choose and group the terms in such a way, that elimination of individual elements takes place. It is necessary to insure here such a notation that no individual possible particular solutions drop out. Thus, if a certain element is excluded by adding a conditional term, this should be reflected in the notation for the result in the form of an equivalence, for example:

$$ ab + \frac{\overline{ab}}{0} = a\frac{1}{b} . \tag{5.26} $$

On the other hand, for the example (5.1) given above, we have

$$ abc + \frac{\overline{abc}}{0} + \frac{\overline{abc}}{0} = a\frac{b}{c} . \tag{5.27} $$

One can obtain from this notation all the particular solutions, the fundamentals of which are ab, ac, abc, and a(b + c).

If the sum is found to contain conditional terms which do not unify with the obligatory terms, they should also be simplified as much as possible, and written as conditional.

Thus, for example

$$ abc + \frac{\overline{abc}}{0} + \frac{\overline{abc}}{0} = ac\frac{b}{1} + \frac{\overline{bc}}{0} . \tag{5.28} $$

As we have seen, simplification of expressions can be obtained by pairwise comparison of all the terms. In some cases one can recommend for convenience   that the terms be arranged in columns /103, 128/, separating the conditional terms by a bar and indicating with arrows which terms are contained in each sum.

Thus, for the expression

$$f = \bar{u}\bar{6}\bar{в} + \bar{u}\bar{6}в + \frac{\bar{u}\bar{6}в}{0} + \frac{\bar{u}\bar{6}в}{0}$$

we obtain
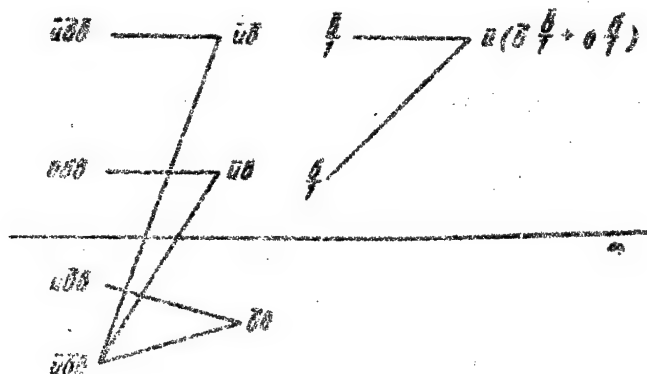
hence

$$f = \bar{u}\left(\bar{6}\,\frac{в}{1} + в\frac{6}{1}\right) + \frac{\bar{6}\,в}{0}.$$

When the number of terms is large, the method of pairwise comparison may be found to be inconvenient, and we shall consider three more general methods.

6. SIMPLIFICATION OF EXPRESSIONS WITH THE AID OF TABLES OF NEIGHBORING CONSTITUENTS

As we have seen, the simplification of expressions is due to

addition of terns corresponding to neighboring constituents. To
facilitate the finding of neighboring constituents, the author
suggests Table 12.

Table 12

№ конст. туннов	Номера соседних конституентов, получающихся при изменении состояния элемента:					
	A(1)	Б(2)	B(4)	Г(8)	Д(16)	E(32)
0	1	2	4	8	16	32
1	0	3	5	9	17	33
2	3	0	6	10	18	34
3	2	1	7	11	19	35
4	5	6	0	12	20	36
5	4	7	1	13	21	37
6	7	4	2	14	22	38
7	6	5	3	15	23	39
8	9	10	12	0	24	40
9	8	11	13	1	25	41
10	11	8	14	2	26	41
11	10	9	15	3	27	43
12	13	14	8	4	28	44
13	12	15	9	5	29	45
14	15	12	10	6	30	46
15	14	13	11	7	31	47
16	17	18	20	24	0	48
17	16	19	21	25	1	49
18	19	16	22	26	2	50
19	18	17	23	27	3	51
20	21	22	16	28	4	52
21	20	23	17	29	5	53
22	23	20	18	30	6	54
23	22	21	19	31	7	55
24	25	26	28	16	8	56
25	24	27	29	17	9	57
26	27	24	30	18	10	58
27	26	25	31	19	11	59
28	29	30	24	20	12	60
29	28	31	25	21	13	61
30	31	28	26	22	14	62
31	30	29	27	23	15	63

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1) Number of constituents, 2) numbers of neighboring consti-
tuents, obtained   upon change in the state of the element.

This table gives the numbers of the neighboring constituents
for all 32 constituents of a network consisting of five (A, B, C, D,
and E) elements (the numbering is made with a base EDCBA). If the
number of elements is greater, the table can be expanded. Thus,
the sixth column contains the numbers of the neighboring constituents,
corresponding to a change in the sixth (F) element. It is seen from
the table, for example, that for the constituent numbered 6, the
neighboring constituents will be numbered 7, 4, 2, 14, 22, 38, etc.
Here the constituent 6 differs from constituent 7 in the state of
element A (since the number 7 is in the column corresponding to this
element), and differs from   constituent 2 in the state of the
element C, etc.

It follows therefore that if, for example, we add to the
constituent numbered 6 ($ab\bar{c}d\bar{e}$) a constituent numbered 7 ($abc\bar{d}\bar{e}$),
then the element a will be excluded in the sum, and upon addition
to constituent 2, the element c will be excluded, i.e., the sums
will be respectively $bc\bar{d}e$ and $a\bar{b}d\bar{e}$, etc.

If we   copy from the table in two lines all the
neighboring constituents (in the sequence in the table), for two
neighboring constituents, then in each column (in addition to the
column in which the numbers of the constituents chosen by us are

located) we obtain one pair each of numbers of constituents, which upon addition with the previously taken constituents will yield the elimination of two elements. Here we eliminate those elements, in the columns of which are located the numbers of the constituents taken as terms.

Thus, for example, for neighboring constituents 6 and 2, we obtain from the table (for the sake of convenience we write over the columns the elements in the state with which they enter into the first term):

	$\bar{a}$	b	c	$\bar{z}$	$\bar{d}$
6	7	4	2	14	22
2	3	0	6	10	18

We see therefore that in addition to the element c, when constituents 3 and 7 are added the element a is elminated (as a result we obtain $\overline{bde}$), when the constituents 0 and 4 are added the element b is excluded ($\overline{ade}$ remains), etc.

We can obtain analogously the numbers of those eight constituents, the addition of which eliminates three elements and among which each constituent has already three neighboring constituents, etc.

Consequently, having for a given circuit an assembly of obligatory and conditional numbers, we can obtain with the aid of the table the final expression. With this, if in any column there are found constituents which do not enter into the

assembly, the element corresponding to this column cannot be excluded.

Thus, in the synthesis of any circuit we can establish the following sequence for the choice and addition of constituents.

1. We make up an assembly of obligatory and conditional numbers, which enter into the general solution for the given circuit.

2. We take one of the obligatory numbers, and from Table 12 we write out the numbers of those neighboring constituents, which can be added to the given one, in the same sequence as in the table. Here we place a bar in the place of the constituents which cannot be added, and single out the individual number (by using italics, for example).

If for a given number no neighboring numbers are found, it means that the expression for this term should be left unchanged. If there is only one neighboring constituent, then as a result of addition to it one eliminates that element, in the column of which is located the number of the added constituent. With this, if the added constituents belongs to the conditional terms, the excluded element is written as a conditional factor. The presence of several neighboring elements indicates that it is possible to exclude one of the elements, corresponding to these neighboring constituents. It may turn out here that it is possible to eliminate simultaneously several elements, if one succeeds in choosing all the necessary terms.

3. We verify the possibility of eliminating two elements. For this purpose we write out the second line for one of the neighboring constituents (with this it is preferable to take the number of the obligatory term). If after this we still find more than one column with numbers still not taken, we take one of these columns (again preferably with a large   amount of numbers of obligatory terms), and write out two additional rows of numbers of the neighboring constituents, corresponding to the two taken numbers. We analogously write out further four, eight, etc. rows, until only one column is left, filled with numbers which have not been taken. In this case the elements corresponding to the filled columns can be excluded, and the sum will contain all the terms, whose numbers are in these columns.

If in the sequence of writing out the rows no filled columns with numbers which have not yet been taken   this indicates that further simplification is impossible.

we find

Thus, if it is found that there exists several columns filled with numbers, and an investigation shows that no further simplification is possible, this means that only one (any) of the elements corresponding to these columns can be eliminated.

We make an analogous operation with all the obligatory terms, which did not enter in the first result (in the first term of the sought answer), and all the obligatory terms already contained there can be considered as conditional. This is repeated with the

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remaining conditional numbers.

For the example (5.1) we shall have

$$f = a\delta s + \frac{\overline{a\delta s}}{0} + \frac{a\overline{\delta s}}{0} = \{7, (5, 3)\}_{AKA.} \tag{5.29}$$

We write out from the table

	u	$\overset{b}{6}$	$\overset{c}{s}$
7	—	5	3
3	—	7	—

We see therefore that because of the conditional terms we can exclude the element b or c, i.e., we can take two solutions — ab or ac — i.e., in the general form we obtain

$$f = a\frac{6}{b}.$$

For the function $f = \left\{ 1, 7, 8, 9, 10, 12 \ (0, 6, 11, 13, 14, 15) \right\}_{DOBR}$, starting, for example, with No. 1, we can write

	a	6	$\overline{6}$	$\overline{z}$
1	0	—	—	9
9	8	11	13	1

It follows therefore that it is possible to exclude the elements a and d, and from among the obligatory terms the sum will contain the constituents 0, 1, 8, and 9. As a result we obtain the circuit $\overline{b}\overline{c}$.

Continuing analogously, we obtain

	a	6	s	$\overline{c}$
7	6	—	—	15
6	7	—	—	14

Consequently, the elements a and d can also be eliminated from the constituent 7.

Inasmuch as this elimination is obtained by adding only the conditional Nos. 6, 14, and 15, we write the result in the

$$6e\,\frac{a}{1}\,\frac{\bar{c}}{1}.$$

For the remaining terms numbered 10 and 12 we obtain

	$\bar{a}$	6	$\bar{e}$	e
10	11	8	14	—
8	9	10	12	0
14	15	12	10	6
12	13	14	8	—

so that as a final result there remains only the element d, and

the numbers 8 and 9, 11, 13, 14, and 15 will again enter.

Thus, all the conditional numbers have been accounted for.

The fact that the foregoing two terms (with numbers 8 and 9)

enter into two members, means that in one of the members they can

be considered conditional. This yields two results

$$f = \bar{6}\,\bar{e} + 6e\,\frac{a}{1}\,\frac{\bar{c}}{1} + e\,\frac{a}{1}$$

and

$$f = \bar{6}\,\bar{e}\,\frac{a\,\bar{c}}{1} + 6e\,\frac{ae}{1} + e = (\bar{6}\,\bar{e} + 6e)\,\frac{a}{1} + e.$$

It follows therefore, that in class π, the simplest circuit

will be

$$f = \bar{6}\,\bar{e} + 6e + e.$$

in
It should be noted that this method certain fundamental parti-

cular solutions may be lost, because the elimination of individual

elements is due to combining both obligatory and conditional terms.

~~Sisux~~ Thus, in the last example the joining of the terms with Nos.

1, 8, and 9 yields the result $\overline{bc}\,(a+d)$. If we add to this the conditional term zero, we obtain $\overline{bc}$.

Thus, $\qquad (1, 8, 9, (0)) = \overline{b}\,\overline{c}\,\dfrac{a+c}{1}$.

and not $\overline{bc}$, as obtained above.

In the case when it is necessary to retain all possible fundamental particular solutions, we must obtain two simplified formulas, corresponding to the maximum f_{max} and minimum f_{min} values of the solution, i.e., with all the conditional numbers and without them, and by comparing these two solutions write down the general solution. Thus, for the last example we obtain

$$f_{min} = \overline{b}\,\overline{c}\,(a+c) + ab\overline{a}\overline{c} + c\overline{a}\,(\overline{b}+\overline{c}),$$

$$f_{max} = \overline{b}\,\overline{c} + bc + c.$$

Hence

$$f = \overline{b}\,\overline{c}\,\frac{a+c}{1} + bc\frac{a\overline{c}}{1} + c\frac{\overline{a}\,(\overline{b}+\overline{c})}{1}.$$

In this case the simplest network of class Π will correspond to the expression f_{max}.

7. METHOD OF SIMPLIFYING EXPRESSIONS WITH THE AID OF MINIMIZING MAPS

Another method of simplification is writing down the expressions with the aid of the so-called minimizing maps /113/.

This method is applicable for solutions which do not have

conditional terms. In the present work we show its use for the simplification of the general solution.

A minimizing map for a network consisting of n relays is a table having 2^n rows, each of which corresponds to one constituent, and $2^n - 1$ columns, corresponding to combinations of the products of variables taken 1, 2, 3, ..., n at a time. In each cell of the table are written down the variables which enter into the given column in those states, in which they enter into the constituent of the given row. Fig. 36 shows a minimizing map for n = 3.

For our case we can recommend the following sequence.

1. We cross out all the rows corresponding to the constituents which do not enter into the given expression.

2. In each column the crossed-out combinations of the variables are crossed out also in the remaining rows.

3. We verify all the uncrossed rows and in those rows which contain only one each of the uncrossed combinations with a minimum number of variables, we frame these combinations. We also frame all such combinations in each column. Each such combination is called "essential" and should enter into the resultant expression.

4. In the uncrossed rows, in which there are no framed combinations, there will be several uncrossed combinations with a minimum number of elements. These combinations are called "free," and one of them may be included in the result. We enclose enclose all the free combinations of each row in a common frame (inside

Fig. 36.

Fig. 37.

the frame there may be also crossed-out combinations, which we shall disregard).

5. The result is written as a sum of all the essential combinations, to which we add the free combinations, written in the form of equivalences.

The resultant expression will correspond to the maximum value of the sought solution.

We then cross out the rows corresponding to the conditional terms, and in analogy with the foregoing find new essential and free combinations (for the sake of clarity we shall circle these) with a minimum value for the sought solution.

From a comparison of both solutions we obtain a simplified notation for the general solution.

In the case when it is necessary to obtain only the simplest expression for the construction of a class $\overline{\overline{\Pi}}$ network, operations 1 and 2 are followed by a choice of combinations from only the rows of obligatory terms, disregarding the rows of the conditional terms.

Fig. 36 shows the determination of the simplified notation of the function

$$ f = \{3, 4, 7, (1, 6)\}_{95A} = a\bar{b}\bar{c} + \bar{a}\,\bar{b}\,c + abc + \frac{\bar{a}\bar{b}\,\bar{c}}{0} + \frac{\bar{a}bc}{0}. \tag{5.30} $$

Crossing out the rows corresponding to constituents 0, 2, 5 and carrying out the choice of the remaining combinations, we

obtain

$$f_{max} = \bar{a}\bar{s} + \bar{a}s + \delta\frac{a}{s}.$$

Then, after crossing out the conditional constituents with numbers
1 and 6

$$f_{min} = a\delta + \bar{a}\,\bar{\delta}s.$$

Comparing these expressions, we obtain

$$f = \bar{a}s\frac{|\bar{\delta}}{1} + \frac{a\delta}{a\bar{s} + \delta\frac{a}{s}} = \bar{a}s\frac{\bar{\delta}}{1} + \frac{a\delta}{a\bar{s} + \delta s}. \qquad (5.31)$$

For simplification in the compilation of the map we can
recommend that in each cell we write not the combination of vari-
ables, but the number corresponding to the sum of the weights of
those variables, which are located in the particular cell without
inversions. By way of an example of such a notation, Fig. 37 shows
the transformation of the expression

$$f = \{1, 7, 8, 9, 10, 12, (0, 6, 11, 13, 14, 15)\}_{ГBEA}.$$

From this map we obtain

$$f_{max} = e + \delta\bar{s} + \delta s,$$

$$f_{min} = a\delta\bar{s} + \bar{a}\,\bar{\delta}e + \bar{a}\,\bar{s}e + a\delta s\bar{e} = a(\bar{\delta}\bar{s} + \delta s\bar{e}) + \bar{a}e(\bar{\delta} + \bar{s}),$$

hence

$$f = e\frac{\bar{a}(\bar{\delta} + \bar{s})}{1} + \left(\bar{\delta}\bar{s} + \delta s\frac{\bar{e}}{1}\right)\frac{a}{1}.$$

When the number of variables is $n > 4$, the use of
minimizing maps becomes difficult because of their cumbersomeness.

When the volume of work is large, it is recommended that the maps be prepared beforehand or that one map be used in connection with tracing paper.

8. METHOD OF SIMPLIFYING THE EXPRESSIONS WITH THE AID OF STENCILS

To simplify the expressions, an original method is proposed for minimization of   notation with the aid of "contact grids" -- special stencils /56/.

For minimization of a function written down in the form of a "coordinate" table (Chapter 2, Section 3), A. Svoboda proposed the use of a special stencil ("grid") for each variable contained in the formula (or its inversion), with cuts in those rows, which correspond to these variables. Thus, for the six variables x_1, x_2, x_3, y_1, y_2, and y_3, the general form of the coordinate table is shown in Fig. 38, and the corresponding stencils on Fig. 39. The same stencil is used for the inversions, but is rotated by 180°. By   making up several stencils and placing them on the tables, we see in the holes the cells of those constituents, which enter into the formula, corresponding to the product of the variables, denoted on the upper and right sides of the stencils.

Thus, Fig. 40 shows the eight constituents corresponding to the expression x_1, x_3, y_2.

The process of minimization reduces to the following. The

	$\bar{y}_1\bar{y}_2\bar{y}_3$	$y_1\bar{y}_2\bar{y}_3$	$\bar{y}_1 y_2\bar{y}_3$	$y_1 y_2\bar{y}_3$	$\bar{y}_1\bar{y}_2 y_3$	$y_1\bar{y}_2 y_3$	$\bar{y}_1 y_2 y_3$	$y_1 y_2 y_3$
$\bar{x}_1\bar{x}_2\bar{x}_3$	0	8	16	24	32	40	48	56
$x_1\bar{x}_2\bar{x}_3$								
$\bar{x}_1 x_2\bar{x}_3$								
$x_1 x_2\bar{x}_3$								
$\bar{x}_1\bar{x}_2 x_3$								
$x_1\bar{x}_2 x_3$								
$\bar{x}_1 x_2 x_3$								
$x_1 x_2 x_3$	7	15	23	31	39	47	55	63

Fig. 38.

Fig. 39.

Fig. 40.

simplified function is written in the table with marking for the obligatory and conditional constituents. The stencils are then chosen such, that when they are placed on the table, one sees through the holes as many obligatory and conditional constituents as possible, and not one of the forbidden ones is seen. The expression read with the aid of the stencils is written as one of the terms, and the designations seen in the holes are crossed out and are henceforth considered as conditional. This process is repeated until all the obligatory constituents are crossed out.

To simplify the process, A. Svoboda recommends that each constituent be assigned a "weight" -- the number of available neighboring obligatory or conditional constituents -- and that the choice begin with the constituents having the maximum weight.

This method can also be used to expand formulas into constituents.

9. Additional Equivalences that Follow from the Operating Sequence of the Relay

In certain cases it becomes necessary to use equivalences /16/ for specified sequences of operation of the relays. These equivalences can be readily obtained by using the concepts of greater and smaller circuits given above. Thus, for example, relay A always operates before relay B, and drops out later, i.e., if the number of states in which the relay A operates is greater

than the number of states in which the relay B operates, and conse-
quantly
$$a \geqslant 6.$$

then, in accordance with (4.23), we obtain

$$\begin{aligned}
a6 &= 6, \\
a + 6 &= a, \\
a + \bar{6} &= 1, \\
\bar{a}6 &= 0.
\end{aligned}$$

$$(5.32)$$

The last equality indicates, in particular, that the state when
the relay B operates and the relay A does not operate does not
exist for the given sequence.

Analogously, if relays A and B never operate simultaneously,
then
$$a \leqslant \bar{6}$$
$$6 \leqslant \bar{a}.$$

or

It follows therefore that

$$\begin{aligned}
a6 &= 0; \quad a + \bar{6} = 1, \\
a\bar{6} &= a; \quad \bar{a} + 6 = \bar{a}, \\
\bar{a}6 &= 6; \quad a + \bar{6} = \bar{6}.
\end{aligned}$$

$$(5.33)$$

If at least one of the relays A or B, always operates, then

$$a \geqslant \bar{6} \quad \text{или} \quad 6 \geqslant \bar{a},$$

hence

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$$\begin{aligned}
\overline{a}\,\overline{\delta} &= 0, & a + \delta &= 1, \\
a\overline{\delta} &= \overline{\delta}, & \overline{a} + \delta &= \delta, \\
\overline{a}\delta &= \overline{a}, & a + \overline{\delta} &= a.
\end{aligned}\right\} \qquad (5.34)$$

Analogous equivalences can be obtained also for circuits with a greater number of relays. Thus, in the case when relays A, B, and C never operate simultaneously, i.e., the following system of inequalities holds

$$a\delta \leqslant \overline{\mathit{s}}, \quad a\mathit{s} \leqslant \overline{\delta} \quad \text{или} \quad \delta\mathit{s} \leqslant \overline{a}.$$

From this, for example, follows

$$\begin{aligned}
a\delta\mathit{s} &= 0, & \overline{a} + \overline{\delta} + \overline{\mathit{s}} &= 1, \\
a\overline{\delta}\mathit{s} &= a\delta, & \overline{a} + \overline{\delta} + \mathit{s} &= \overline{a} + \overline{\delta},
\end{aligned}\right\} \qquad (5.35)$$

etc.

The use of these equivalences will be ~~is~~ illustrated when an example of the synthesis of a network for transmitting a number from the relay RA, RB, RC, RD, RE, and RF by means of a counting network of a relay register of commercial equipment /122, 129/ to four fixing relays over circuits I, II, III, and IV.

The operation of the network is specified in Table 13.

It follows from the table that, for example, circuit I should be closed when the number 1, 3, 5, 7, or 9 is chosen, i.e., the formula of the circuit of this relay will be (we discard the letter R for simplicity)

$$f_1 = a\overline{\delta\mathit{s}z\partial}e + \overline{a}\overline{\delta}\mathit{s}\,\overline{z\partial}e + \overline{a}\overline{\delta}\mathit{s}z\partial e + a\overline{\delta}\mathit{s}z\overline{\partial}e + \overline{a\delta}\,\mathit{s}\,z\partial e.$$

Table 13

1) Набираемая цифра	2) Счетные реле						3) Исполнительные цепи			
	РА RA	РБ RB	РВ RC	РГ RD	РД RE	РЕ RF	I	II	III	IV
1	+						+		+	
2		+						+		+
3			+				+	+		
4				+					+	+
5					+		+			+
6						+		+		
7	+					+	+	+	+	
8		+				+		+		
9			+			+	+			
0				+		+				+

1) Dialed number, 2) counting relay, 3) actuating circuits.

Since according to the operating conditions for the network, only one of the relays A, B, C, D, or E can operate, and relay E cannot operate simultaneously with F, then according to (5.33)

and consequently

$$\overline{\overline{a}\overline{б}\overline{в}\overline{г}\overline{д}} = a, \quad \overline{\overline{a}\overline{б}\overline{в}\overline{г}\overline{д}} = в \text{ и } \overline{\overline{a}\overline{б}\overline{в}\overline{г}\overline{д}\overline{е}} = д,$$

$$f_1 = a\overline{e} + в\overline{e} + д + ae + ве = a + в + д.$$

Analogously we obtain (the detailed construction is shown in /18/, pp 66 — 68).

$$f_{II} = (б + в)\overline{e} + \overline{б}\overline{в}\overline{г}e;$$
$$f_{III} = a + гe + бe;$$
$$f_{IV} = б\overline{e} + г + д.$$

The corresponding network (Fig. 41a) has nine less springs than the same network in the existing register (Fig. 41b).
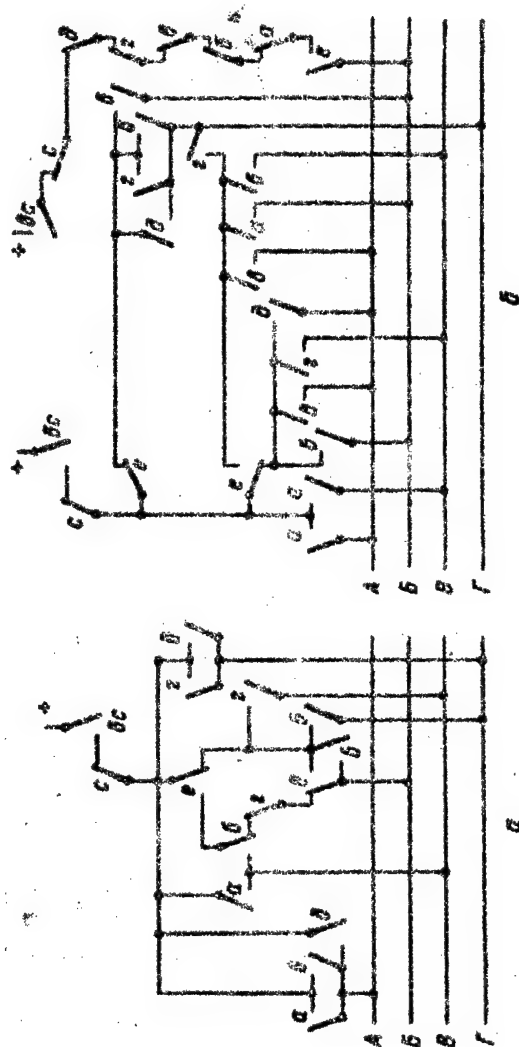
FIG. 41.

Chapter 6

ASSEMBLIES OF NUMBERS AND THEIR TRANSFORMATIONS

1. Basic Definitions

As shown in Chapter 2, the operating conditions of a contact network can be written in the form of assemblies of numbers of those states, in which the given circuits should or could be closed. With this, the number of state is taken to mean the sum of the weights $q_i = 2^{i-1}$ of those relays of the network, which operate in the given state.

The sequence of arrangement of the numbers within the obligatory and conditional parts of the assembly is of no significance, as follows from the correctness of the commutative law for contact networks, and therefore for the sake of convenience we shall write the numbers in the assemblies in increasing order. Thus, a circuit which should be closed in states with numbers 7, 0, and 2 can be closed in states with numbers 5 and 1, will be written in the form of the assembly $f = \left\{ 0, 2, 7(1, 5) \right\}$.

An assembly of obligatory and conditional numbers corresponds uniquely to the notation for the general solution in the form of the sum of obligatory and conditional constituents:

$$f = k_{\eta_1} + k_{\eta_2} + \dots + k_{\eta_r} + \frac{k_{\mu_1}}{0} + \frac{k_{\mu_2}}{0} + \dots + \frac{k_{\mu_s}}{0} =$$
$$= \{\eta_1, \eta_2, \dots, \eta_r, (\mu_1, \mu_2, \dots, \mu_s)\}_B = \{N, (M)\}_B.$$

(6.1)

where: η_i — obligatory numbers $(i = 1, 2, \dots, r)$;

μ_j — conditional numbers $(j = 1, 2, \dots, s)$;

k_{η_i} and k_{μ_j} — constituents corresponding to these

numbers;

N — assembly of obligatory numbers;

M — assembly of conditional numbers;

r — number of obligatory terms;

s — number of conditional terms;

B — base.

As will be shown later, assemblies of numbers serve as the basis of the graphical construction of contact networks (Chapter 7). We shall dwell therefore in greater detail on operations with these assemblies, but we first introduce several supplementary definitions.

We note first of all that in the general case the numbers of states, and consequently the assemblies also, change for a given network depending on the order with which the weights (numbers) are assigned to individual relays of the network. The chosen order of assigning weights to the relays will be called the base and we shall agree to write it in the form of a list of the relays with decreasing weight. Thus, for example, when the base is written DCBA this means that the relay D has a weight 8, relay C

has a weight 4, relay B has a weight 2 and relay A has a weight 1.
We shall agree, when necessary, to write down the base in the form of
an index for the assembly after closing the curly brackets. In the
case when the sequence of assignment of the weights is immaterial,
and it is important merely to note the number of relays in the network,
n, this number will be written in the form of an index instead of
the base.

An assembly which contains all the 2^n numbers of states, 0,
1, 2, ..., $2^n - 1$, will be called complete. If all the numbers are
in addition obligatory, such an assembly will be called absolutely
complete. This assembly corresponds to a constantly closed circuit,
i.e.,

$$\{0,1,\ldots,2^n - 1\} \equiv 1.$$

(6.2)

An assembly which contains not a single obligatory number
will be called empty. If in this case there are likewise no con-
ditional numbers, such an assembly, consisting constantly of an
open circuit, will be called absolutely empty

$$\{- (-)\} \equiv 0.$$

(6.3)

States in which the given circuit cannot be closed will be
called forbidden. The number of such states will be $2^n - (r + s)$.
Starting with the definition of the conditional numbers, we can
say that in states with these numbers the circuit can be opened.
In other words, each circuit can be characterized also by an
assembly of numbers, in which it should be or could be opened

$$[\nu_1, \nu_2, \ldots, \nu_p, (\mu_1, \mu_2, \ldots, \mu_s)]. \qquad (6.4)$$

where ν_i are the forbidden numbers ($i = 1, 2, \ldots; p = 2^n - r - s$).

In analogy with the statements made in Chapter 5 regarding general solutions, we can say that the notation for any circuit in the form

$$f = \{N, (M)\} \qquad (6.5)$$

corresponds to the inequality

$$\{N\} \leqslant f \leqslant \{N, M\}, \qquad (6.6)$$

i.e., an assembly consists of only obligatory numbers is the least value of the circuit, written by formula (6.5), an assembly that includes all the obligatory and conditional numbers is the largest value of this circuit, i.e.,

$$\left. \begin{array}{l} f_{min} = \{N\}, \\ f_{max} = \{N, M\}. \end{array} \right\} \qquad (6.7)$$

For the sake of clarity we shall represent this graphically in the form of so-called Euler circles (Fig. 42), where the rectangle is the volume of all the 2^n states, the internal circle is the smallest value of the function, and the external circuit is the maximum value. The boundary of the sought function should be located in the ring formed by these two circles.

2. Basic Operations with Assemblies

As with algebraic expressions, one can carry out mathematical

operations analogous to operations of algebraic contact networks with assemblies.

Before proceeding to these operations, we note that the same number cannot simultaneously be both obligatory and conditional in the same assembly. If any number α is simultaneously in both parts of the assembly, it must be excluded from the conditional part, as follows from the repetition law

$$\{a, (a)\} = k_\alpha + \frac{k_a}{0} = \frac{k_a + k_a}{k_1 + 0} = k_1 = \{a\}. \qquad (6.8)$$

In the formulation of individual operations with assemblies, we shall bear in mind that all this pertains to assemblies with one and the same base. The question of transformations of assemblies connected with the change in the base will be considered in Sections 4 to 6 of the present chapter.

Considering, as is customary in the theory of contact networks, that addition corresponds to a parallel connection of circuits, and multiplication corresponds to a series connection, we can establish the following rules for operations with assemblies.

A sum of several assemblies gives an assembly in which the obligatory terms are the obligatory numbers of all the terms of the assemblies without repetition, while the conditional ones are those conditional numbers of the terms of the assemblies, which were not included among the obligatory ones (Fig. 43a).

This follows from the fact that when adding functions expanded into constituents, the sum will contain all the constituents

of the terms.

Thus, for example,

$$\{1, 3, 5, (2, 6)\} + \{4,5,6(2,3,7)\} =$$

$$= k_1 + k_3 + k_5 + \frac{k_2}{0} + \frac{k_6}{0} + k_6 + k_4 + k_5 + k_4 + \frac{k_2}{0} + \frac{k_3}{0} + \frac{k_7}{0}$$

$$= k_1 + k_3 + k_4 + k_5 + k_6 + \frac{k_2}{0} + \frac{k_7}{0} = \{1, 3, 4, 5, 6, (2,7)\}.$$

The product of several assemblies gives an assembly in which the obligatory are those numbers contained in the obligatory parts of all the multiplied assemblies, while the conditional ones are the numbers which are contained in all the multiplied assemblies, but which are conditional in at least one of them, (Fig. 43b).

The same follows from the multiplication of functions expanded in constituents with allowance for the fact that $k_i.k_j = 0$ when $i \neq j$ and $k_\beta \cdot \frac{k_\beta}{0} = \frac{k_\beta}{0}$.

Thus, for example

$$\{1, 3, 5, (2, 6)\} \cdot \{4, 5, 6, (2, 3, 7)\} =$$

$$= \left(k_1 + k_3 + k_5 + \frac{k_2}{0} + \frac{k_6}{0} \right) \left(k_1 + k_5 + k_6 + \frac{k_2}{0} + \frac{k_3}{0} + \frac{k_7}{0} \right) =$$

$$= k_5 + \frac{k_2}{0} + \frac{k_3}{0} + \frac{k_6}{0} = \{5, (2, 3, 6)\}.$$

If we denote by 1 the presence of some number in the obligatory part, by 1/2 its presence in the conditional part, and by zero its absence from the assembly, the position of this number in the resultant assembly, after addition or multiplication, can be characterized by the following tables (in the first column and the first row are indicated the values of the numbers in the initial

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assemblies, and at the intersection of the corresponding rows and columns are the values of the number in the resultant assembly);

	addition				multiplication		
	1	½	0		1	½	0
1	1	1	1		1	½	0
½	1	½	½		½	¼	0
0	1	½	0		0	0	0

In the case of inversion, one obtains the circuit whose structural admittance is the inverse of the admittance of the initial circuit, i.e., a circuit which should be closed in those states when the initial circuit is open, and vice versa. It follows therefore that upon inversion the obligatory part of the new assembly will contain all the numbers which are complements of the inverted assembly (i.e., the forbidden numbers), while the conditional parts will contain the conditional numbers of the inverted assembly (Fig. 43c),i.e., the presence of any particular number in the resultant assembly is determined upon inversion by the following table:

0	1/2	1
1	1/2	0

For example, if $f = 1, 3(2, 6)$, then

$$\bar{f} = \overline{\{1, 3, (2, 6)\}} = (0, 4, 5, 7, (2, 6)).$$

The sum of the initial function and its inversion gives
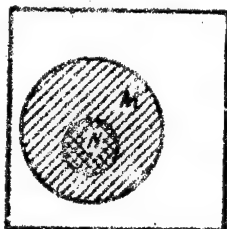
Fig. 42.
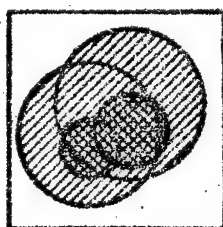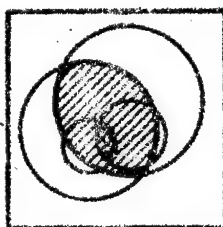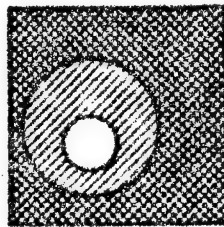
Fig. 43.
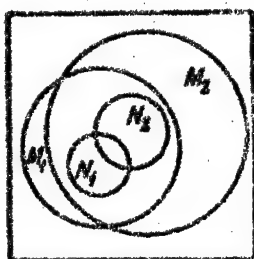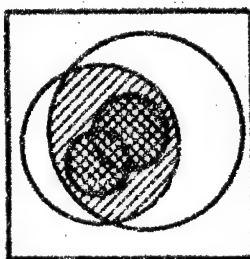
Fig. 44.

a complete assembly, and the product gives an empty assembly:

$$f + \bar{f} = \{0, 1, 3, 4, 5, 7, (2, 6)\};$$

$$f \cdot \bar{f} = \{(2, 6)\}.$$

Addition, multiplication, and inversion of assemblies with obligatory and conditional numbers corresponds to analogous operations in three-valued logic /1 to 4/, in which one half if taken to mean "not determined."

3. COINCIDING ASSEMBLIES AND UNIFICATION

We now introduce another concept and operation, which we did not encounter in the algebraic notation for circuits, but which are very important in the graphical method of construction of contact networks.

Assemblies of which the obligatory number of any assembly are completely contained in each of the remaining assemblies, although among the conditional ones there, will be called coincident. For example, the assemblies $\{0, 2, 7(1, 5)\}$, $\{0, 1, 7, (2, 5, 6)\}$, and $\{1, 2, (0, 5, 7)\}$ will coincide.

From the definition of the coinciding assemblies it follows that the assemblies

$$f_1 = \{N_1, (M_1)\},$$
$$f_2 = \{N_2, (M_2)\},$$
$$\cdots \cdots \cdots \cdots$$
$$\cdots \cdots \cdots \cdots$$
$$f_k = \{N_k, (M_k)\}$$

$$(6.9)$$

will coincide if the following inequalities are satisfied (Fig. 44a)

or
$$(N_i) < (N_j, M_j)$$
$$l_{i\,min} \leqslant l_{j\,max}$$

(6.10)

for all the assemblies f_i and f_j $(i \neq j)$.

From the definition of coinciding assemblies it follows that among the obligatory numbers of each of these assemblies there are no forbidden numbers, pertaining to circuits corresponding to other of these assemblies. Thus, for the circuits given above, the numbers 3, 4, 6; 3, 4, and 3, 4, 6 will be forbidden, and these, as we see, do not enter into the obligatory parts of any of the assemblies. For contact circuits this means that it is possible to find a circuit which will satisfy each of the coinciding circuits. In other words, the circuits written down in terms of the coinciding assemblies can be unified and replaced by one common circuit, which should not close in all the states with forbidden numbers of each of the joined circuits.

The operation of finding such a unified circuit will be called unification and will be noted by the symbol $\bigcirc$.

Unification can be considered as finding the general solution for all particular solutions, satisfying simultaneously all the general solutions that are to be unified.

The unified circuit $f^{\bigcirc} = \left\{ N^{\bigcirc} , (M^{\bigcirc}) \right\}$ for the coinciding circuits f_1, f_2, ..., f_k should satisfy each of the circuits to be unified, i.e., the following system of inequalities

should be satisfied:

$$
\left.\begin{array}{l}
f_{1_{min}} \leqslant f^O \leqslant f_{1_{max}}, \\
f_{2_{min}} \leqslant f^O \leqslant f_{2_{max}}, \\
\quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
\quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
f_{k_{min}} \leqslant f^O < f_{k_{max}}.
\end{array}\right\}
$$

(6.11)

It follows from the left part of the system (6.11) that the minimum value f_{min} of the unified circuit f^O should be

$$
f^O_{min} = f_{1_{min}} + f_{min} + \ldots + f_{k_{min}} = \sum_{i=1}^{k} f_{i_{min}}.
$$

(6.12)

The maximum value f^O_{max}, on the basis of the right half of system (6.11), will be

$$
f^O_{max} = f_{1_{max}} \cdot f_{2_{max}} \ldots f_{k_{max}} = \prod_{i=1}^{k} f_{i_{max}}.
$$

(6.13)

In other words, the unified circuit should satisfy the inequality

$$
\sum_{i=1}^{k} f_{i_{min}} < f^O \leqslant \prod_{i=1}^{k} f_{i_{max}}.
$$

(6.14)

This leads to a rule for obtaining a unified assembly, in which the obligatory numbers are all the obligatory numbers of the unified assemblies, and the conditional ones are the conditional numbers which are repeated in each of these assemblies.

In other words, in unification the obligatory part equals the sum of the obligatory parts, while the conditional one is the product of the conditional parts of the unified assemblies.

The action of unification can be characterized by means of the graph
of Fig. 44b or the following table:

	1	½	0
1	1	1	—
½	1	½	0
0	—	0	0

Thus, for example,

$$\{0, 2, 4, (3, 6)\} \cdot \{0, 3, 4, (2, 5, 6)\} \cdot \{2, 3, (0, 4, 6)\} = \{0, 2, 3, 4, (6)\}.$$

4. TRANSFORMATION OF ASSEMBLIES UPON CHANGE OF BASE

When the base is changed the numbers of the states also change,
and consequently the assemblies and the networks can also change.
The change in the base can be due either to transposition (rearrange-
ment) of its elements, or by changing the number of relays in the
network.

Upon transposition of the base, the sequence with which the
relays are numbered changes, and consequently their weights change.
Since the number of elements remains the same in this case, the
number of different sequences of numbering (number of different
bases) in a network consisting of n relays will be the number of
permutations of n, i.e., n!. In each transposition part of the
number changes in accordance with a definite law.

Thus, for example, if we use the base ABC instead of the base

CBA, corresponding to a change in the weights of the relays C and A, the following transformation of numbers takes place

$$\frac{B\ Б\ A}{A\ Б\ B} \rightarrow \frac{0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7}{0,\ 4,\ 2,\ 6,\ 1,\ 5,\ 3,\ 7}.$$

If we change, however, to the base ACB, the transformation of the numbers will be

$$\frac{B\ Б\ A}{A\ B\ Б} \rightarrow \frac{0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7}{0,\ 4,\ 1,\ 5,\ 2,\ 6,\ 3,\ 7}.$$

In these transpositions of the base, the assembly $\{1,\ 3,\ 5\ (2,\ 6)\}_{CBA}$ will be correspondingly changed to

$$\{1,\ 3,\ 5,\ (2,\ 6)\}_{BБA} = \{4,\ 5,\ 6,\ (2,\ 3)\}_{AБB} = \{4,\ 5,\ 6,\ (1,\ 3)\}_{ABБ}.$$

Without dwelling on the general laws of change in numbers upon transposition of the base, we note that the smallest (zero) and the largest $(2^n - 1)$ numbers do not change in any transposition.

The situation will be different   as regards change of numbers if the number of relays in the network is changed.

Thus, if there is a certain circuit f made up of contacts of n relays, and we wish to add to the network one more relay X with weight $q_{n+1} = 2^n$ without changing the circuit, we must add to the circuit f the parallel-connected make and break contacts of the relay X. In other words, the new circuit can be represented in the form of two parallel circuits, differing only in the fact that one of them contains in series the break contact of relay X, while the second contains the make contact. This is equivalent to saying

that each number α , contained in the assembly of the circuit f in the old base, changes into two numbers when the relay X is added to the base: α , corresponding to the state in which the relay X does not operate, and $\alpha + q_{n+V1}$ corresponding to the state in which the relay X operates.

Thus, the assembly $\{0, 2, 5(3, 6)\}_{CBA}$ becomes the assembly $\{0, 2, 5, 8,10, 13 (3, 6, 11, 14)\}_{\overset{X}{X}CBA}$ when a relay of weight 8 is added.

In the general case, when m relays are added to the network, each number α goes into 2^n numbers by adding to it all possible sums of the weights q_{n+1}, q_{n+2}, ..., q_{N+m} of the added relays, i.e., it produces the series

$$\alpha, \ \alpha + q_{n+1}, \ \alpha + q_{n+2}, \ldots, \ \alpha + q_{n+m}, \ \alpha + q_{n+1} + q_{n+2}, \ldots,$$
$$\alpha + q_{n+1} + q_{n+2} + \ldots + q_{n+m}.$$

Thus, when two relays X and Y with weights 8 and 16 are added to the network, the assembly $\{0, 2, 5(3, 6)\}_{CBA}$ becomes the assembly
$$\{ 0, 2, 5, 8, 10, 13, 16, 18,$$
$$21, 24, 26, 29, (3, 6, 11, 14, 19, 22, 27, 30)\}_{YXBBA}.$$

If the added relays are assigned the least weights 1, 2, ..., 2_{n-1}, then the weights of the basic relays are correspondingly increased by a factor 2^n, and consequently, each number α will go into the series
$$\alpha \cdot 2^m, \ \alpha \cdot 2^m + 1, \ \alpha \cdot 2^m + 2, \ldots, \ \alpha \cdot 2^m + 2^m - 1.$$

Thus, for example, in the network containing two relays with

base YX there exist a circuit with the assembly $\{1, (2)\}$, then upon addition of three relays A, B, and C with weights 1, 2, and 4 the weights of the relays Y and X should be increased by a factor $2^3 = 8$, and consequently the assembly $\{1,(2)\}_{XY}$ goes into the assembly $\{8, 9, 10, 11, 12, 13, 14, 15(16, 17, 18, 19, 20, 21, 22, 23)\}_{YXCBA}$.

Using these rules, we can obtain a general numbering for the case of joining of several circuits with different bases into a common network.

Thus, for example, if one connects in series the aforementioned circuits $\{0, 2, 5(3, 6)\}_{CBA}$ and $\{1(2)\}_{YX}$, then the common circuit will be determined by the assembly which is the product of the two assemblies, reduced to the base YXCBA, i.e., $\{8, 10, 13(11, 14, 16, 18, 19, 21, 22)\}_{YXCBA}$.

It is easy to verify, using the rule for the transformation of numbers when changing the base and multiplying assemblies, that in the case of a series connection of circuits with assemblies

and
$$
\left.\begin{aligned}
f_1 &= \{\eta_1, \eta_2, \ldots, \eta_s, (\mu_1, \mu_2, \ldots, \mu_s)\}_{x_n, \ldots, x_1} \\
f_2 &= \{\alpha_1, \alpha_2, \ldots, \alpha_{r'}, (\beta_1, \beta_2, \ldots, \beta_{s'})\}_{y_m, \ldots, y_1}
\end{aligned}\right\}
\qquad (6.15)
$$

in the case of the new base $X_n, \ldots, X_2, X_1, Y_m, \ldots, Y_2, Y_1$ each of the obligatory numbers η_k of the assembly f_1 will yield r' obligatory numbers, determined from the formula

$$
\eta_k \cdot 2^m + \alpha_i \quad (i = 1, 2, \ldots, r').
$$

and s' conditional numbers

$$\eta_{ij} \cdot 2^m + \beta_j \, (j = 1, 2, \dots, s').$$

Each conditional number μ_χ of the assembly f_1 goes into r' + s'
conditional numbers

and
$$\mu_i \cdot 2^m + \alpha_i$$
$$\mu_i \cdot 2^m + \beta_j$$

Thus, the obligatory part of the assembly of the overall cir-
cuit will have r.r' obligatory numbers and rs' + s(r' + s') condi-
tional numbers:

$$f = f_1 f_2 = \left\{ \sum_{i}^{r} \sum_{i'}^{r'} (\eta_{ij} \cdot 2^m + \alpha_i), \left(\sum_{i=1}^{r} \sum_{j=1}^{s'} (\eta_{ij} \cdot 2^m + \beta_j), \sum_{i=1}^{s} \sum_{i=1}^{r'} (\mu_i \cdot 2^m + \alpha_i), \right. \right.$$
$$\left. \left. \sum_{i}^{s} \sum_{j=1}^{s'} (\mu_i \cdot 2^m + \beta_j) \right) \right\}_{x_n, \dots, y_1} \tag{6.16}$$

Thus, for example, when circuits with assemblies $\left\{ 0, 2, 5, \right.$
$\left. (3, 6) \right\}_{CBA}$ and $\left\{ 1(2) \right\}_{YX}$ we obtain with the new base CBAYX
the following assembly

$$\{1, 9, 21, (2, 10, 13, 14, 22, 25, 26)\}_{CBAYX}.$$

In the particular case, when the assembly f_2 contains only
one obligatory number i, the assembly of the overall circuit
contains r obligatory and s conditional numbers, where each number
γ of the assembly f_1 goes into the number

$$\gamma' = \gamma 2^m + i. \tag{6.17}$$

We now examine the change that will be produced in the
assemblies, if some relay is eliminated from the network. As is well

known, on the basis of the expansion formula (4.15a):

$$f(a, b, \ldots, n) = af(1, b, \ldots, n) + \bar{a}f(0, b, \ldots, n),$$

we can separate out of any circuit a transfer contact of one of the relays.

In other words, any circuit can be represented as a parallel connection of two circuits, one of which is connected in series with the make circuit and the other with the break circuit of any one of the relays of the network. If the circuit is specified by means of an assembly, then to separate out the transfer contact the assembly must be broken up into two, one containing the numbers of the states in which the separated relay does not operate, and the other in which   this relay does operate.

Thus, when the contact of a relay with weight 1 is separated, the first assembly will contain all the even and the second all the odd numbers. For a relay with the maximum weight q_n, the first assembly will include all the numbers from zero to $q_n - 1$, and the second the numbers from the relay weight q_n and higher.

In the general case the relay with weight q_i does not operate in states with numbers from $2\xi q_i$ to $(2\xi + 1)q_i - 1$, and operates in the states numbered $(2\xi + 1)q_i$ to $(2\xi + 2)q_i - 1$, where $\xi =$ 0, 1, 2, $\ldots$, $2^{n-1}/q_i - 1$. The list of the numbers of states in which the relays with weights from 1 to 16 operate and do not operate are given in Table 14 and in Fig. 45.

Thus, the circuit specified by the assembly $\left\{ N, (M) \right\}$ (Fig.46a)
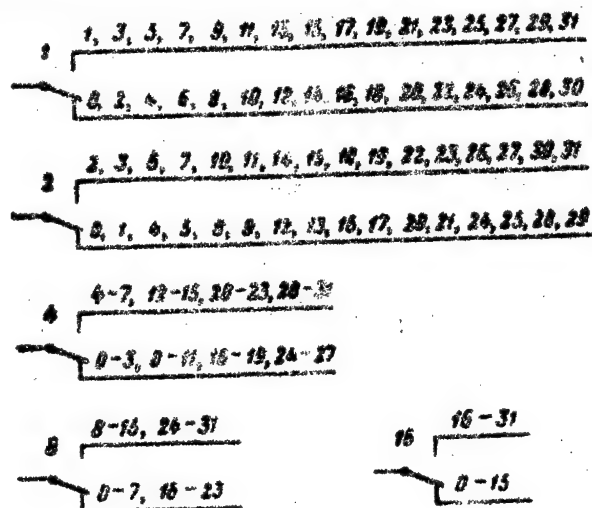
Fig. 45.
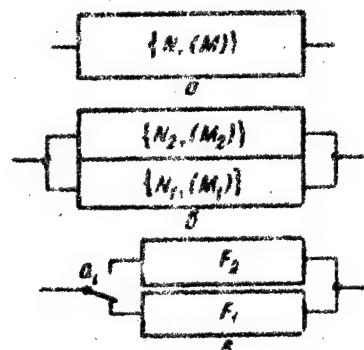
Fig. 46.

can be represented as a parallel connection of two circuits with assemblies $\{N_1, (M_1)\}$ and $\{N_2 (M_2)\}$, of which the first contains only circuits closed in those states, in which the relay A_i with weight q_i operates, and the second those in which it does not operate (Fig. 46b). If we now separate the transfer contact of relay A_i, there remain two circuits f_1 and f_2 (Fig. 46c), made up of contacts of the remaining relays, i.e., having a new base, differing from the base of the initial circuit by the absence of the relay A_i.

If in the new base all the elements are placed in the same sequence as in the base of the initial network, i.e., the numbers from A_1 to A_{a-1} remain, and the numbers of the relays from $A_i + 1$ to A_n are reduced by one, then the numbers of the sets $\{N_1 (M_1)\}$ and $\{N_2, (M_2)\}$ should be changed in accordance with the following rule. The numbers of the assembly $\{N_1, (M_1)\}$ of a circuit connected with the break contact is reduced by the amount ξq_i, while the numbers of the second assembly, $\{N_2, (M_2)\}$, are reduced by an amount $(\xi + 1)q_i$, where ξ is taken to be the same as in the determination of the numbers that enter into the assemblies of both circuits. The transformed numbers are shown in italics in Table 14.

Thus, for example, if in the circuit with assembly $\{0, 2, 5, (3, 6)\}_{CBA}$ we separate the contact of the relay B with weight 2, we obtain two new assemblies

$$\{N_1, (M_1)\} = \{0,5\}_{BБA} \text{ и } \{N_2, (M_2)\} = \{2, (3, 6)\}_{BБA}.$$

Reducing to the new base CA, we obtain

$$f_1 = \{0,3\}_{B\Lambda} \quad \text{и} \quad f_2 = \{0, (1, 2)\}_{BA}.$$

It is most convenient to bring out the contact of the relay with the maximum weight $q_n = 2^{n-1}$. In this case the circuit connected in series with the break contact will have an assembly with numbers lesser than the weight of the relay, while the second, connected in series with the make contact, will have numbers starting with the weight of the relay and higher, from which the weight of the relay is subtracted.

Table 14

i	Вес, q_i	Номера состояний, в которых реле A_i
		не работает
1	1	0,2,4,6,8,10,12,14,16,18,20,22,24,26,28,30
2	2	0,1,4,5,8,9,12,13,16,17,20,21,24,25,28,29
3	4	0,1,2,3,8,9,10,11,16,17,18,19,24,25,26,27
4	8	0,1,2,3,4,5,6,7,16,17,18,19,20,21,22,23
5	16	0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15
		0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15
		работает
1	1	1,3,5,7,9,11,13,15,17,19,21,23,25,27,29,31
2	2	2,3,6,7,10,11,14,15,18,19,22,23,26,27,30,31
3	4	4,5,6,7,12,13,14,15,20,21,22,23,28,29,30,31
4	8	8,9,10,11,12,13,14,15,24,25,26,27,28,29,30,31
5	16	16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31
		0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15

1) Weight, q_i, 2) number of states in which the relay A_i, 3) does not operate, 4) operates.

Thus, for the circuit indicated above $\left\{0, 2, 5(3, 6)\right\}_{CBA}$ when separating the contact of the relay C with weight 4 we obtain

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$\left\{N_1(M_1)\right\} = \left\{0,\ 2(3)\right\}_{CBA}$ and $\left\{N_2(\overline{M_2})\right\} = \left\{5, (6)\right\}_{CBA}$. Going over to the new base, we have $f_1 = \left\{0,\ 2(3)\right\}_{BA}$ and $f_2 = \left\{1(2)\right\}_{BA}$.

In the second case, if the contact of relay with the smallest weight, 1, is taken out, the new assembly for the circuit connected with the break contact will consist of even numbers of the initial assembly, divided by two, and for the circuit connected to the make contact it will consist of odd numbers, reduced by unity and also divided by two.

Consequently, for the circuit $\left\{0,\ \mathbf{2},\ 5(3,\ 6)\right\}_{CBA}$ we obtain upon separating the contact of the relay A, $f_1 = \left\{0,\ 1, (3)\right\}_{CB}$ and $f_2 = \left\{2, (1)\right\}_{CB}$.

We note still another factor. It if obvious that in the case when the circuits f_1 and f_2, which remain after taking out the transfer contact of any relay, are equal, this contact can be completely eliminated.

Thus, if there exists a circuit with an assembly $\left\{1,\ 3,\ 9,\ 11(2,\ 10)\right\}_{CBA}$, then upon separation of the transfer contact of relay C we obtain $f_1 = f_2 = \left\{1,\ 3, (2)\right\}_{BA}$, i.e., the relay C can be eliminated.

However, the elimination of the contact is permissible also when the circuits are not equal, but have coinciding assemblies. In this case the circuit f_1 and f_2 can be replaced by a circuit with the unified assembly, since the latter satisfies both circuits.

Thus, for example, if a circuit with assembly $\{1, 10(2, 3, 8, \overline{9})\}_{CB}$ is specified, then after excluding the transfer contact of relay C we obtain $f_1 = \{1, 2(3)\}_{BA}$ and $f_2 = \{2(0, 1)\}_{BA}$. The circuits f_1 and f_2 have coinciding assemblies and can be replaced by a circuit with unified assembly $\{1, 2\}_{BA}$, which corresponds to the assembly $\{1, 2, 9, 10\}_{CBA}$, which satisfies the given circuit.

The separation of the transfer contact serves as the basis of the graphic method of construction of contact networks.

Chapter 7

GRAPHIC METHOD OF CONSTRUCTING CONTACT NETWORKS

1. METHOD OF CASCADES

Algebraic methods make it possible in most cases to construct contact networks of class $\overline{\overline{\prod}}$ to satisfy given expressions. However, as is well known, the introduction of bridge elements in many cases makes it possible to obtain simpler networks.

One of the methods of going over from the algebraic expression to a bridge network is the so-called method of cascades /45/, developed by G. N. Povarov /41/. This method is a generalization of the method of synthesis of parallel-series two-terminal networks with the aid of contact trees.

The essence of this method consists of the following. If there are specified k functions of n variables: $f_1(x_1, x_2, \ldots, x_n)$, ..., $f_k(x_1, x_2, \ldots, x_n)$, characterizing the structural admittance of k outputs to one input, then for each of these functions $f_i(x_1, x_2, \ldots, x_n)$ one can separate a transfer contact of one of the relays, for example, x_1

$$f_i(x_1, x_2, \ldots, x_n) = x_1 f_i(1, x_2, \ldots, x_n) + \bar{x}_1 f_i(0, x_2, \ldots, x_n).$$

The functions remaining after separating the variable x_1 can

be written as new functions k_1 of $n - 1$ variable, i.e.,

$$h_1(x_2, x_3, \ldots, x_n), \ldots, h_{k_1}(x_2, x_3, \ldots, x_n),$$

where $k_1 \quad 2k$ (since for different i the functions h can be identical).
If we now take a $(1, k_1)$-pole H (Fig. 47), which realizes all these functions h, then each circuit can be represented as consisting of a transfer contact $x_1 -- \overline{x}_1$, the leads of which are connected to the corresponding outputs of the $(1, k_1)$-pole, realizing the functions $f_1(1, x_2, \ldots, x_n)$ and $f_1(0, x_2, \ldots, x_n)$.

Inasmuch as the transfer contact $x_1 - \overline{x}_1$ is a contact pyramid (tree) with one contact, then by virtue of the separability of pyramids, their outputs corresponding to different i can be joined to one and the same output of the $(1, k_1)$-pole H.

Thus, a change over takes place from the $(1, k)$-pole with n relay contacts to a $(1, k_1)$-pole with $n - 1$ relay contacts. Continuing the construction, we arrive at a new $(1, k_2)$-pole made up of contacts of $n - 2$ relays, etc., until the network contains contacts of all the relays.

We note that the series introduction of transfer contacts leads to the situation whereby at each stage of construction, after the contacts of any relay are introduced into the network, the resultant multipole will always be isolating as seen by the outputs of the network.

The process of constructing by the cascade method will be explained with an example of the construction of the network of a

(1, 2)-pole made up of contacts of four relays, specified by the following structural formulas:

$$f_1 = x_1(x_3 x_4 + \overline{x}_3 \overline{x}_3) + \overline{x}_1 \overline{x}_3 x_4;$$
$$f_2 = x_2 x_3 \overline{x}_4 + \overline{x}_3 x_4.$$

To simplify the notation, the circuit from any node of the network numbered i   which are written in the form of this number, included in parentheses (i), i.e., in the given case $f_1 = (1)$, and $f_2 = (2)$.

We continue the notation in the form of Table 15, with successive separation of contacts x_1, x_2, etc.

Table 15

$(1) = x_1 (3) + \overline{x}_1 (4)$	$(3) = x_3 x_4 + \overline{x}_3 \overline{x}_4$
	$(4) = \overline{x}_3 x_4$
$(2) = x_1 (5) + \overline{x}_1 (6)$	$(5) = x_2 x_3 \overline{x}_4 + \overline{x}_3 x_4$
	$(6) = x_2 x_3 \overline{x}_4 + \overline{x}_3 x_4 = (5)$

$(3) = x_2 (7) + \overline{x}_2 (8)$	$(7) = x_3 x_4 + \overline{x}_3 \overline{x}_4$
	$(8) = (7)$
$(4) = x_2 (9) + \overline{x}_2 (10)$	$(9) = \overline{x}_3 x_4$
	$(10) = (9)$
$(5) = x_2(11) + \overline{x}_2 (12)$	$(11) = x_3 \overline{x}_4$
	$(12) = x_4$

$(7) = x_3(13) + \overline{x}_3 (14)$	$(13) = x_4 = (12)$
	$(14) = \overline{x}_4$
$(9) = \overline{x}_3(15)$	$(15) = x_4 = (12)$
$(11) = x_3(16)$	$(16) = \overline{x}_4 = (14)$

The corresponding network is shown in Fig. 48.

In his paper, G. N. Povarov /44/ indicates that the complexity of the network will depend on the sequence of arrangement of the variables (i.e., on the sequence with which the transfer contacts are separated out). He then proceeds to deduce that if certain functions of a realizable sequence depend essentially on less than n variables, then the variables on which all the functions depend essentially are best placed in the last places, near the common input of the synthesized (1, k)-pole network.

2. GRAPHICAL METHOD OF CONSTRUCTION OF SYMMETRICAL NETWORKS

Somewhat later, developing the cascade method, G. N. Povarov /44/ proposed a graphic method for constructing contacts (1, k)-poles for symmetrical circuits, specified in terms of working numbers, i.e., numbers indicating for how many working relays should the circuit be closed.

With this, as in the method of cascades, the transfer contacts of the relays are drawn successively. To construct, one spots along a vertical line k points, corresponding to the outputs, and working numbers are assigned to each point. Then one draws adjoining to each point the transfer contact of one of the relays (it is immaterial which, since the complexity of the symmetrical network is independent of the sequence); the point connected to the break contact is assigned all the working numbers

with the exception of n (number of the relay of the network), while the point connected to the make contact are assigned the numbers greater than zero, reduced by unity. Then the points with identical assemblies of numbers are joined, and the points without numbers are erased. If near any point there are located all the numbers from zero to n - 1, such a point is joined to the common input.

The transfer contact of the next relay is drawn joined to the remaining points, and in analogy with the preceding, the working numbers are changed, where n is taken to be the number of the remaining relays. The process is repeated until the contacts of all relays are introduced into the network. With this, the last relay will always have one transfer contact.

Fig. 49 shows an example of the construction of a (1, 3)-pole (k = 3) made up of contacts of four (n = 4) relays for circuits having the following working numbers: 0, 2; 1, 3, and 1, 3, 4. Simultaneously with proposing the method, G. N. Povarov has shown that the process is applicable not only for symmetrical networks, but for quasi-symmetrical ones, for which the working number represents a sum of several weights, ascribed to individual relays of the network.

The author of the present work has later shown that all the networks are in this sense quasi-symmetrical, if relays numbered $i = 1, 2, \ldots, n$ are assigned weights 2^{i-1}, and the working numbers are taken to be the numbers of the states during which the

Fig. 47.

Fig. 48.

Fig. 49.

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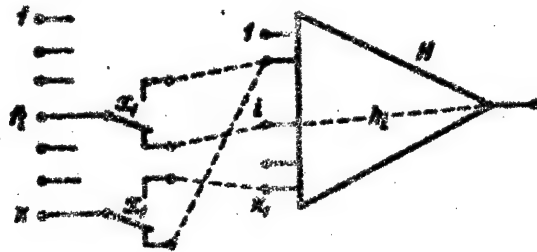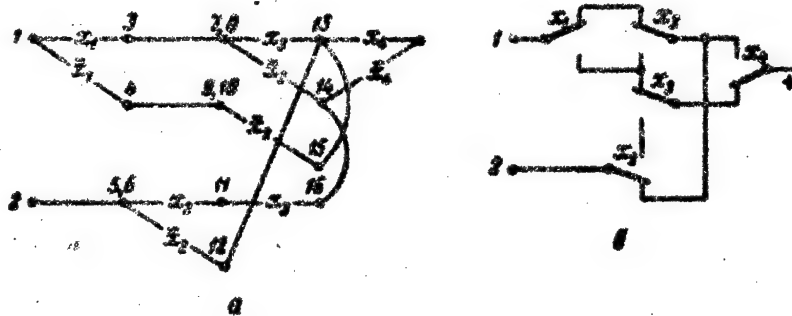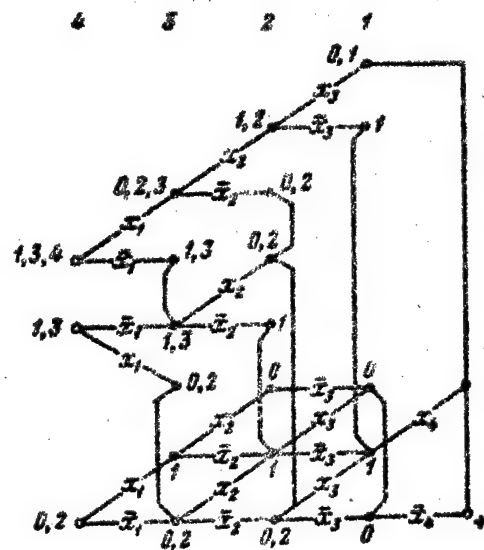
circuit should be closed.

3. GRAPHICAL METHOD OF CONSTRUCTING CONTACT (1, k)-POLE

The graphical method of constructing contact (1, k)-poles makes it possible to construct contact networks without analytical notation and transformations. In the construction one obtains automatically simplifications due to the presence of indifferent and unused states. One of the features of this method is that the constructed network is obtained both of class Π or class H, either planar or non-planar depending on which network is simpler, something that cannot be obtained in general by analytical methods.

Experience in employing the graphical method shows that the constructed networks, particularly networks of contact-making multipoles, are obtained in most cases simpler than those networks constructed by other methods.

The only limitation which the graphic method imposes on the networks is an ordered arrangement of the contacts of each relay of the network in definite sections. However, this limits the number of different versions of the network to the value n!, which is an undoubted advantage of the method. Deviation from the foregoing limitation can yield in principle additional simplifications of the network, but this problem has not yet been investigated. Simplicity of operations and identity of the actions performed in the construction of the network by the graphical method makes it

possible to employ it for mechanization of the process of synthesis
of relay networks, as will be shown in Chapter 11.

Inasmuch as the graphic method in its initial form was a
graphic interpretation of the method of cascades with addition of
those simplifications, which can be obtained in the allowance for the
conditional (indifferent and unused) states, we shall not stop
specially in this section on the justification of any particular con-
struction, but will go to a direct development of the method in that
form, in which one can recommend it for practical engineering use.

For greater clarity we shall develop this method first as
applied to the construction of contact $(1, k)$-poles. We shall not
consider two-poles, since they are particular cases of $(1, k)$-poles.
We note that in the construction of contact networks one should
tend to a maximum unification of the circuits of the network into
one $(1, k)$-pole. Most frequently the common point of connection
of the circuits is one of the poles of the battery.

Thus, by way of initial data for the graphical construction
of a contact $(1, k)$-pole network made up of contacts of n relays,
we have k assemblies, corresponding to circuits from each of the
k outputs to the common input.

To compile the network, we note on the left, along a vertical
line, all the outputs and write near each one of them the corres-
ponding assemblies, and for simplification we shall leave out
the curly brackets and the base. We first compile the common input,

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which we shall denote by plus.

If the assemblies include coinciding ones, the corresponding points will join and the unified assembly will be written out at the common point. In the case when in a complete assembly (all the numbers from zero to $2^n - 1$) correspond to any point, we join this point on a straight line with the point "$+$." To all the remaining points draw on the right the moving springs of the transfer contact groups of the relay having the greatest weight, $q_n = 2^{n-1}$. The point connected with the break contact we assign the numbers less than 2^{n-1}, and the point joined with the make contact we assign the remaining numbers, reduced by the weight of the relay $q_n = 2^{n-1}$. In this case the conditional numbers are written in parentheses, as before.

We then make a comparison of all the assemblies in the following sequence.

1) We see whether there are any coinciding assemblies at the leads from the make and break contacts of one group, and if such assemblies are found, we erase the entire contact group and replace it by a single point, to which we assign the unified assembly.

2) We verify whether there are complete assemblies, and if such are found the corresponding points are joined with the input.

3) We erase the springs with empty assemblies.

4) We unify points with coinciding assemblies, and write down the unified assembly at the junction.

We note that because of the presence of conditional numbers, the solution may be non-unique. Thus, if there are three points with assemblies: a) -- $\left\{1, 5(2, 3, 7)\right\}$; b) -- $\left\{1, 2(4, 5, 7)\right\}$ and c) -- $\left\{3, 5(0, 1, 4)\right\}$, we can join either the points (a) and (b) with a common assembly $\left\{1, 2, 5(7)\right\}$, or (a) and (c) with an assembly $\left\{1, 3, 5\right\}$.

The newly placed points can be considered as the outputs of a new $(1, k_1)$-pole made up of contacts of $n - 1$ relays. To these points we attach the spring contacts of the transfer groups of a relay with weight $q_{n-1} = 2^{n-2}$, and analogously, write down the assemblies of numbers and where possible unify. These points represent outputs of a new $(1, k_2)$-pole made up of the contacts of $n - 2$ relays. We continue the construction until it is complete. The network must then be redrawn, joining where possible all the remaining closing and all the remaining make and break contacts into transfer groups.

Fig. 50a shows an example of the construction of a network for the circuits of the intermediate relays A and B of a binary pulse divider, specified by assemblies $f_A = \left\{1, 2, 3, 6\right\}_{BAT}$ and $f_B = \left\{2, 5, 6, 7\right\}_{BAT}$. In this network, as in the preceding one, we shall represent for the sake of clarity the erased contacts by dots, and the erased numbers will be crossed out. In the redrawn form, the network is shown in Fig. 50b.

We note that in the last relay with weight 1 there is always

⌐one transfer group, and we shall therefore   draw it in this manner.⌐

On Figs. 51a and b is shown the construction for two bases of

the circuits X_0, X_1, X_2 from the register of the commercial equipment

/122, 129/, specified by the following assemblies:

$$X_0 = \{1,\ 2,\ 4,\ 7,\ 9, (6,\ 11,\ 13,\ 14,\ 15)\}_{\Gamma B E A}$$
$$X_1 = \{1,\ 7,\ 8,\ 9,\ 10,\ 12, (0,\ 6,\ 11,\ 13,\ 14,\ 15)\}_{\Gamma B E A}$$
$$X_2 = \{2,\ 3,\ 4,\ 5, (0,\ 6,\ 11,\ 13,\ 14,\ 15)\}_{\Gamma B E A}$$

$$(7.1)$$

$$(7.1)$$

If the base is ABCD, these assemblies will be

$$X_0 = \{2,\ 4,\ 8,\ 9,\ 14, (6,\ 7,\ 11,\ 13,\ 15)\}_{A B B \Gamma}$$
$$X_1 = \{1,\ 3,\ 5,\ 8,\ 9,\ 14, (0,\ 6,\ 7,\ 11,\ 13,\ 15)\}_{A B B \Gamma}$$
$$X_2 = \{2,\ 4,\ 10,\ 12, (0,\ 6,\ 7,\ 11,\ 13,\ 15)\}_{A B B \Gamma}$$

$$(7.2)$$

The networks constructed by this method are in general bridge

networks. They have that distinguishing feature, however, that in

any vertical section on the output side (to the left) they are

isolating, i.e., under no conditions can two points of one vertical

section be found joined   at relay contacts, placed between the

given section and the outputs. This is explained by the fact that

by virtue of the construction itself, any such circuit will contain at

least two different contacts of one relay. As a result, false cir-

cuits,   which   present a danger in the construction of

bridge circuits, cannot appear.

The method developed in the present section has the following

shortcomings: in the construction of the network one always intro-

duces a transfer group and the possibility of elimination of one

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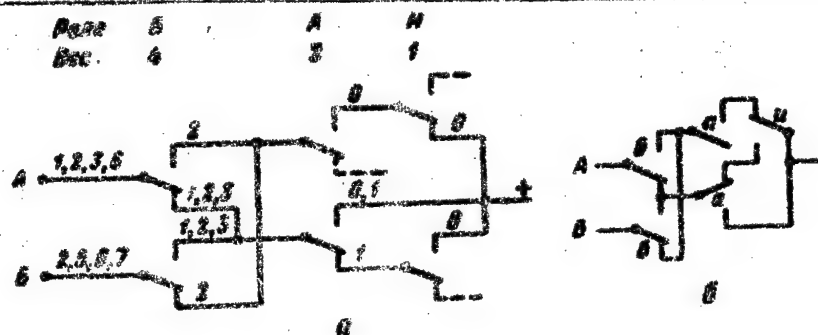
Fig. 50.
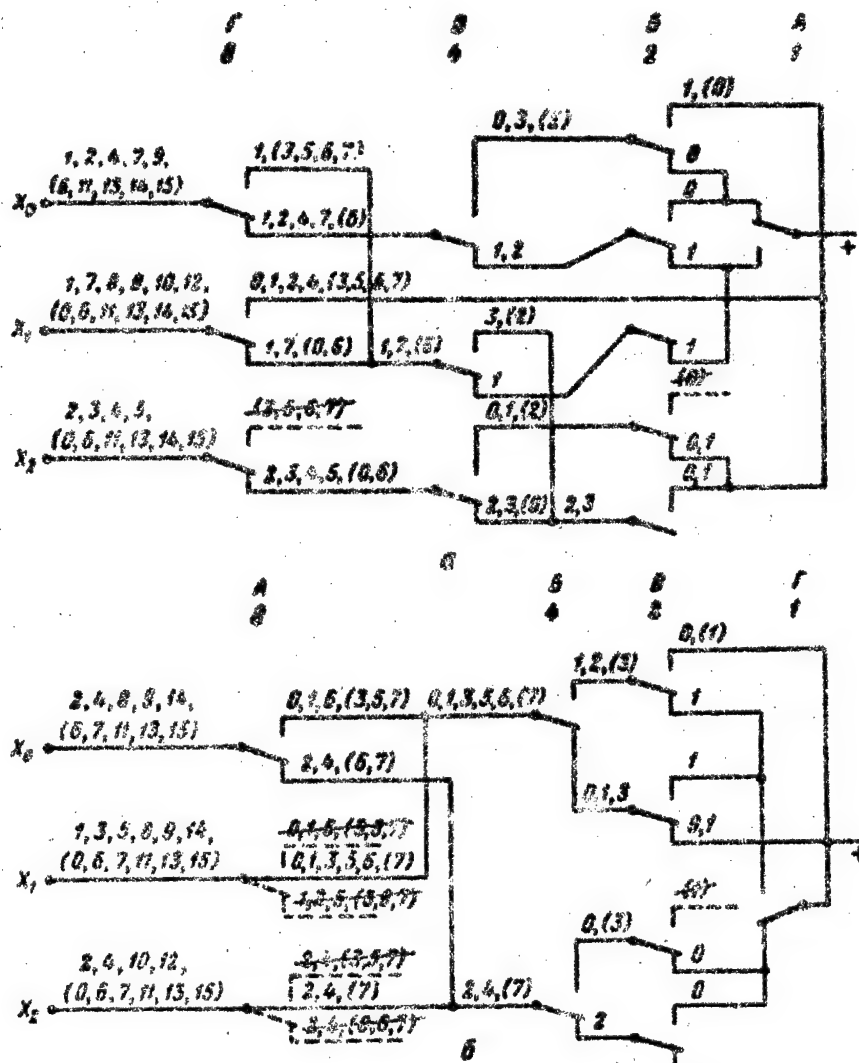
Fig. 51.

of the contacts of the group, resulting from the equivalents (4.13a):

$$a + \bar{a}x = a + x.$$

is not taken into account.

The presence of transfer groups makes the network isolating and makes it possible to carry out unification but, as indicated by M. A. Gavrilov /69/, by far not all the possibilities of circuit unification afforded by the creation of bridge networks are used. This shortcoming has been eliminated by the author by using the so-called direct lead outs in the construction.

4. USE OF DIRECT LEAD OUTS

In some cases the number of contacts in the network can be reduced if one uses so-called direct lead-outs in the construction. Such lead-outs can be made when the assemblies of both contacts of one switching group do not coincide, but identical obligatory numbers are present. In this case one can make a direct lead-out at the contact (Fig. 52), the assembly of which will contain the common numbers of both assemblies. The only obligatory numbers left at the contacts are those which were not included in the direct-lead-out assembly, while the numbers contained in this last assembly are written as conditional. This corresponds to the following transformation

$$x(a+b) + \bar{x}(a+c) = a + x\left(b + \frac{a}{0}\right) + \bar{x}\left(c + \frac{a}{0}\right). \qquad (7.3)$$

Thus, for example (Fig. 52a), if the make contact

corresponds to the assembly $\left\{1, \overset{1}{5}, (2, 3, 7)\right\}$, and the break contact corresponds to the assembly $\left\{0, 2, 5(4, 7)\right\}$, we can make a direct lead-out with assembly $\left\{5, (2, 7)\right\}$ or $\left\{(2, 5, (7)\right\}$; in the latter case we leave respectively at the contacts the assemblies $\left\{1, (2, 3, 5, 7)\right\}$ and $\left\{0, (2, 4, 5, 7)\right\}$ (Fig. 52b and c).

The use of direct leads is particularly effective if the obligatory numbers of one contact are contained in their entirety in the assembly of the second contact. In this case one contact of the transfer group can be crossed out, corresponding to the transformation

$$x(a + b) + \bar{x}a = a + x\left(b + \frac{a}{0}\right). \qquad (7.4)$$

For example, if the make contact of the relay with weight 8 has an assembly $\left\{1, 4, 5(2, 3, 7)\right\}$, and the make contact has an assembly $\left\{2, 5, (0, 4, 7)\right\}$, the latter can be replaced by a direct lead with assembly $\left\{2, 5, (4, 7)\right\}$, leaving   at the make contact the assembly $\left\{1, 4, (2, 3, 5, 7)\right\}$, or else with assembly $\left\{2, 4, 5, (7)\right\}$ with the assembly $\left\{1, (2, 3, 4, 5, 7)\right\}$ left at the make contact (Fig. 53).

The advisability of making direct leads must be verified every time, for sometimes they may complicate the network.

A network with direct leads violates the condition of isolation, and in some cases individual points in the vertical cross section of the network may be found to be joined to the input through contacts of the following relays and through the direct
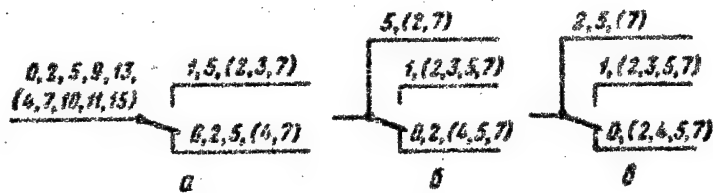
Fig. 52.
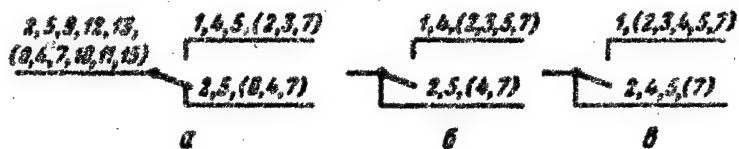
Fig. 53.
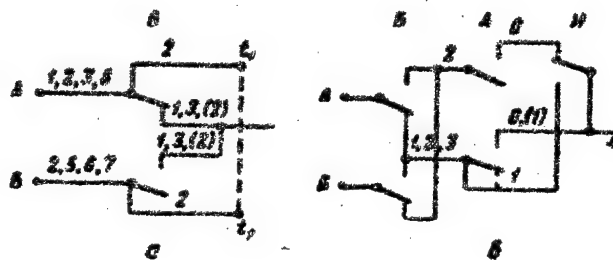
Fig. 54.
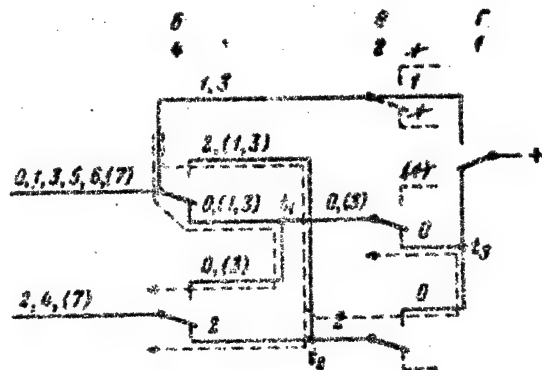
Fig. 55.
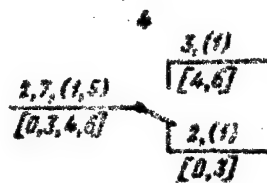
Fig. 56.
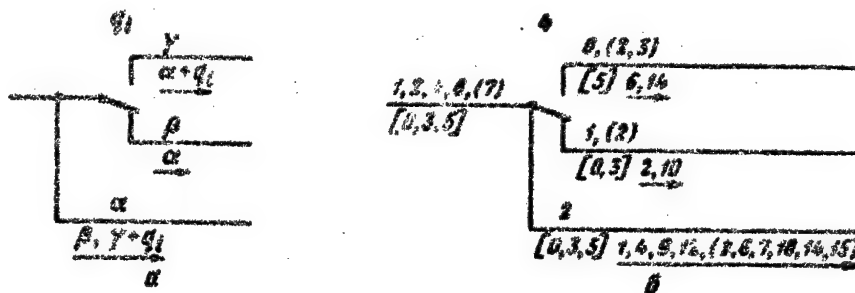
Fig. 57.

leads, connected to these contacts. When joining such points to other circuits, the latter may be found to be joined to the input along round-about circuits, and this may disturb the operation of the network.

Thus, for example, in the circuit of Fig. 50, direct leads with assembly $\{2\}$ can be made at both contacts of the relay B, while the remaining contacts will have assemblies $\{1, 3, (2)\}$, as shown in Fig. 54a. However, one cannot join the direct leads (points t_1 and t_2), since the circuits of relays A and B will be permanently interconnected. At the same time, in the network of Fig. 50, the direct lead from the contact of relay A with assembly $\{1\}$ does not introduce any changes in the operation of the network (Fig. 54b). Analogously, in the network of Fig. 51b one can make a direct lead with assembly $\{1, 3,\}$ at the upper contact of relay B, as a result of which we obtain the network shown in Fig. 55 (only the right half of the network of Fig. 51b is shown.

We see thus that in the presence of direct leads one cannot always join points with coinciding assemblies, since false circuits may appear.

As a result it may turn out that in a network with direct leads one must place on the relay with the least weight more than one transfer contact group.

5. DETECTION OF FALSE CIRCUITS

False circuits may appear when direct leads are produced in and the circuits which are connected to these leads are further unified. In some cases, as observed for example in the networks of Figs. 54 and 55, one can readily verify whether the direct lead and the unifying of the wires are permissible or not. In the network of Fig. 54b, the circuits joined to the direct lead are joined only at the point "plus" so that no false circuit can be produced. In the network of Fig. 55, the direct-lead circuit is not connected to anything, and when other wires are joined at the points t_1, t_2, and t_3, there are no false circuits (dotted in Fig. 55), produced, since they will always contain different contacts of the relay B (if joined at the points t_1 and t_2) or C (joining at the point t_3).

In complicated networks, it may be difficult to trace the appearance of false circuits, we therefore propose a method which makes it possible to establish directly under what unifications do false circuits appear, and when they do not. On the other hand, not every false circuit, even if it is found to connect the input with one of the outputs, disturbs the operation of the network. Thus, if some circuit is closed in states corresponding to the numbers entering in the assembly of the given output, such a false circuit is permissible.

To determine in which cases the joining is permissible and

in which it is not, we shall assign to each output, with the exception of the assemblies indicated above, also forbidden numbers.

For points joined with the output through a make contact of any relay A_i with weight q_i, the forbidden numbers will be only the forbidden numbers of the output, corresponding to the states during which relay A_i does not operate. For points joined through a make [sic] contact, the numbers will correspond to states during which the relay A_i operates. These numbers can be obtained with the aid of Table 14 and Fig. 45.

We shall agree to write out the forbidden numbers under the corresponding point and include them in square brackets, or else write them with a colored pencil. In this case the numbers for all points of the network will be referred to the base of the entire network.

Thus, for the output with assembly $\left\{ 2,\ 7,\ (7,\ 5) \right\}$ the forbidden numbers will be zero, 3, 4, and 6, while at the outputs of the contact of relay with weight 4, there will be written correspondingly the forbidden numbers zero, 3 and 4, 6, as shown in Fig. 56.

In the presence of a direct lead, the latter should be assigned all the forbidden numbers. When joining several points, the common point should be assigned the forbidden numbers of all the joined points, i.e., the assemblies of forbidden numbers add up in the joining. Thus, when constructing a network, along with

successive assignment of a new assembly of numbers to each new

point, we can assign to the same point the forbidden numbers, i.e.,

we indicate the numbers of those states of the network, at which

the given point must not have a connection with the input to the

made up (i, k)-pole.

We now see in what states the appearance of false circuits

through direct leads is possible. It is easy to verify that if

in the assembly of the direct lead, made at the contact of the relay

A_i with weight q_i, there is an obligatory number α (corresponding to

the fact that this lead will be joined to the input of the circuit

at the state with number α of those relays, whose weights are less

than q_i), then in the state, with this number the lead of the break

contact will be found joined to the input in the case when the relay

A_i does not operate, i.e., in the state with the same number α.

The lead of the make contact will be joined to the input through

the direct lead in the state with number $\alpha + q_i$ (Fig. 57a).

On the other hand, the obligatory number β, which enters

into the assembly of the break contact, will produce a false cir-

cuit to the direct lead in the state with the same number β,

while the obligatory number γ, which enters into the assembly of

the make contact, will produce a false circuit with number $\gamma + q_i$.

All this pertains to the conditional numbers, . the only

difference being that it is not known beforehand whether in the

state corresponding to the conditional number the circuit will be

connected to the input or not.

However, the numbers obtained in the above method will pertain to a base which includes the relays with weight q_i and less. If we refer the numbers to the base of the entire network, each number will go into 2^{n-i} numbers, determined by the states of those relays, whose weights are greater than q_i, i.e., into the series

$$x, \; x + q_{i+1}; \; x + q_{i+2}; \ldots; \; x + q_n; \; x + q_{i+1} + q_{i+2}; \ldots; \; x + q_{i+1} + \ldots + q_n.$$

These numbers indicate in what states of the network a given point will be (or can be, if the number is conditional) connected to the input to a direct lead bypassing the main circuit. We shall agree to call these numbers bypassed numbers and write them down along with the forbidden numbers under the point to which they pertain, underlining them with an arrow, as shown in Fig. 57b. For the contact of a relay with weight 4 in a network made up of four relays (n = 4).

In the further construction of the network, the bypassed numbers are transformed in accordance with the same law as the forbidden numbers.

Thus, in the general case one can assign to each point, with the exception of the fundamental assembly, forbidden and bypassed numbers. It is obvious that one and the same number cannot be included simultaneously in the list of forbidden and bypassed numbers of any point, since the forbidden number indicates that in this state the point must not be connected to the input, whereas

the bypassed number indicates that the point in this state will or can be connected with the input. This leads to the rule, that the joining of two points with coinciding assemblies is impossible, if among the bypassed numbers includes forbidden number of the second point.

False circuits, which can be formed through two or more direct leads, are analogously detected. For this purpose, after joining, the bypassed numbers are transferred to the preceding points and a verification is made whether any disagreements are found.

Fig. 58a shows the construction of a network of a binary divider for the number of pulses with the forbidden and bypassed numbers written out. From an examination of the network one can see that the joining of the points t_1 and t_2 is impossible, since the numbers 1, 3 and 5, 7 are bypass numbers for one of these and forbidden for the other. Analogously, we cannot join the points t_3 and t_4. On the other hand, the points t'_1 and t'_2, as well as all leads past the contacts of the relay I, can be joined.

One can verify the validity of the joining at the points t_1, t_2, and t_3 in the circuit of Fig. 55, if the forbidden and bypassed numbers are assigned to all points.

In practice, there is no need for writing out the forbidden and bypassed numbers for the entire network. It is recommended that this be done only for those points, for which it is necessary to ascertain the possibility of joining when the points are

connected with contacts, at which direct leads exist. In addition, there is no need of making a special verification if the joined points are connected to different contacts of one and the same relays, as is observed, in the joining of Fig. 55. Nor is there any need of verifying the possibility of joining at the input point.

If the direct leads are made at some relay A_1 which does not have the maximum weight, it is enough to start to write out both the forbidden and the bypassed numbers with points lying directly ahead of the contact of the relay A, as shown, for example, in Fig. 55, which is part of the network of Fig. 51b.

In some cases the bypassed numbers can be used to produce main circuits, thereby simplifying the network. This, however, calls for additional investigations.

6. Elimination of False Circuits

In the case when it is found that the joining of circuits is impossible, the construction of the network can be continued without joining, as was done, for example, in Fig. 58. The resultant network is shown in Fig. 59a.

In addition, the following possibilities exist for preventing the appearance of false circuits:

1) Elimination of the direct lead, through which the false circuit is produced. For the network of Fig. 58a, this means a change over to the network of Fig. 54b.
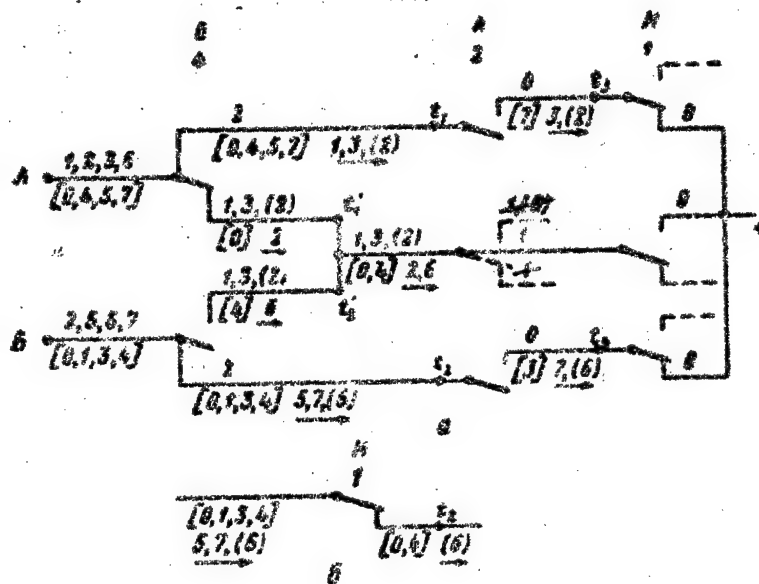
Fig. 58.
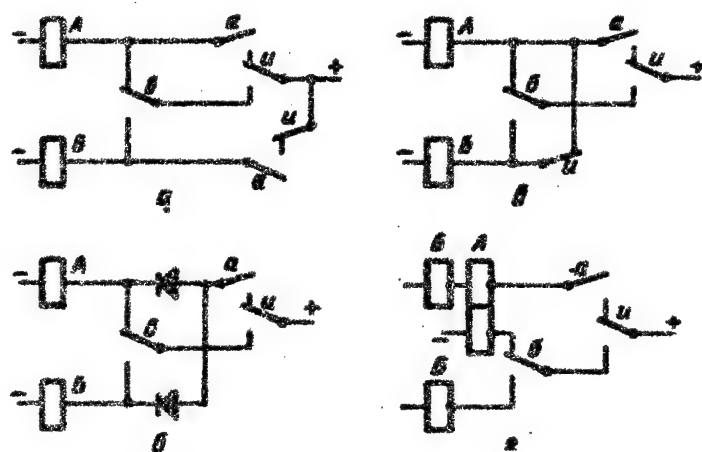
Fig. 59.

2) Introduce rectifier elements in the joined circuits (in the case when the plus of the current source is connected to the input, the rectifier should be connected in opposition to the arrow indicating the direction of the bypassed circuit). Upon introduction of rectifiers ahead of the points t_1 and t_2 in Fig. 58a, we obtain the network of Fig. 59b.

3) Introduce into the joined circuits a contact of one of the relays, which would not contradict the main circuit, but which would eliminate the false circuit. For the network of Fig. 58a, such a contact can be the break contact of relay I (with weight 1). If this contact is connected, for example, ahead of point t_2 (Fig. 58b), a possibility appears of unifying points t_1 and t_2 and the network assumes the form shown in Fig. 59c.

4) Isolate the direct lead as a separate output. This is possible only when the circuits act on the relay and additional windings can be placed on the relay. This method can be particularly effective in the case when the direct lead is made at the contact of the relay with the greatest weight, as is observed, for example, in the network of Fig. 58a. By separating both direct leads and introducing second windings on relays A and B, we obtain the network of Fig. 59d.

Finally, in the case when the joining is impossible because of a circuit produced by a conditional number (the bypassed number is in brackets), this circuit can be eliminated by eliminating

the   particular conditional number from the corresponding assem-
bly.

There are still no rules for indicating which of the measures
gives the most successful solution.

7. CONSTRUCTION OF   ISOLATING NETWORKS

In some cases it becomes necessary to construct an isolating
(1, k)-pole network, i.e., one in which there can be no connection
between the   outputs, with the exception of the inputs.

If we consider the process of the synthesis of contact networks
the
by graphical method, we can see that in the general case the
constructed   circuits will not have the isolating property, for
when two points with coinciding assemblies are joined, a circuit
may be produced connecting different outputs. It follows therefore
that to obtain an isolating network one must not join points if they
have in some state of the network simultaneous connections to
different outputs of the network.

It can be shown that in the case when there are no identical
numbers in assemblies of different outputs, the   network
must be isolating. If there are identical numbers, a circuit may
be produced between corresponding outputs in addition to the
common input.

In order to use the graphic method for the construction of
isolating networks, we employ the following procedure. To each

number which enters into the assemblies of several outputs, we assign a certain symbol (dot, circle, bar, etc.) and different symbols are assigned to different numbers. Upon further construction these symbols will be drawn on the newly obtained numbers, obtained from the marked number. If coinciding assemblies appear now, then in the case when these assemblies are found to have numbers with identical symbols, such points cannot be joined. If assemblies wherein some number is assigned different symbols are joined, both symbols must be assigned to this number in the joined assembly. Any assembly can be joined at the point of input.

Using the method of construction of isolating (1, k)-pole networks, one can construct "two-sided" normal networks, as shown in Fig. 60. The construction of such networks makes it possible to reduce the total number of contacts, but this is possible only for the case when the formulas for each of the relays are functionally separable /36/ and can be expressed in the form of a product of two functions of different variables.

$$f_i(a_1, a_2, \ldots, a_n) = f'_i(a_1, a_2, \ldots, a_i) f''_i(a_{i+1}, \ldots, a_n).$$

8. CONSTRUCTION OF A CONTACT (p, k)-POLE NETWORK

The graphic method can also be extended to the construction of contact multiple-pole networks with p inputs and k outputs, under the condition that in this (p, k)-pole not one of the outputs can be connected simultaneously to more than one input.
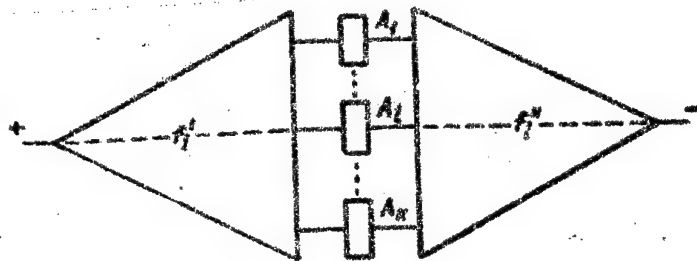
Fig. 60.
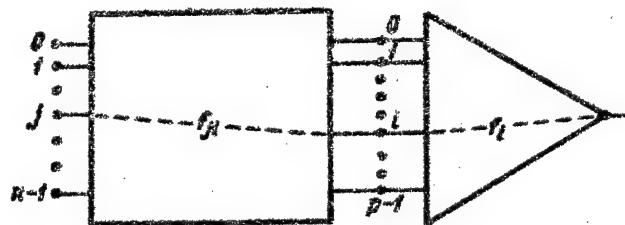
Fig. 61.

To compile the network, the circuits between the inputs and the outputs should be specified in the form of assemblies f_{ji} from each output with number j ($j = 0, 1, 2, ..., k - 1$) to each input with number i ($i = 0, 1, 2, ..., p - 1$). Thus, the total number of assemblies (including empty ones) should be pk.

In order for the network to satisfy the foregoing requirement that there be no simultaneous connection between an output and several inputs, it is sufficient that in the assemblies pertaining to one output (with identical values of the index j) no identical obligatory numbers be found.

To construct such a (p, k)-pole network from the contacts of n relays, we convert it into a $(1, k)$-pole of $n + m$ relays, where n is the smallest integer satisfying the inequality $2^n \geqslant p$. We now assume that of the contacts of m relays there is constructed a $(1, p)$-pole in such a way, that to each output of this network corresponds an assembly f_i of one number, equal to the number i of the output. If we join the inputs of the specified (p, k)-pole with the like outputs of the $(1, p)$-pole (Fig. 61), then the resultant $(1, k)$-pole in the circuit from the output j to the common input through the point i will consist of the series connected circuits f_{ji} (of the (p, k)-pole and the circuit f_i of the $(1, p)$-pole. Since the assembly of the circuit f_i consists of only one obligatory number, i.e., $f_i = \left\{ i \right\}$, then, in accordance with the statements made in Chapter 6, Section 4, the number
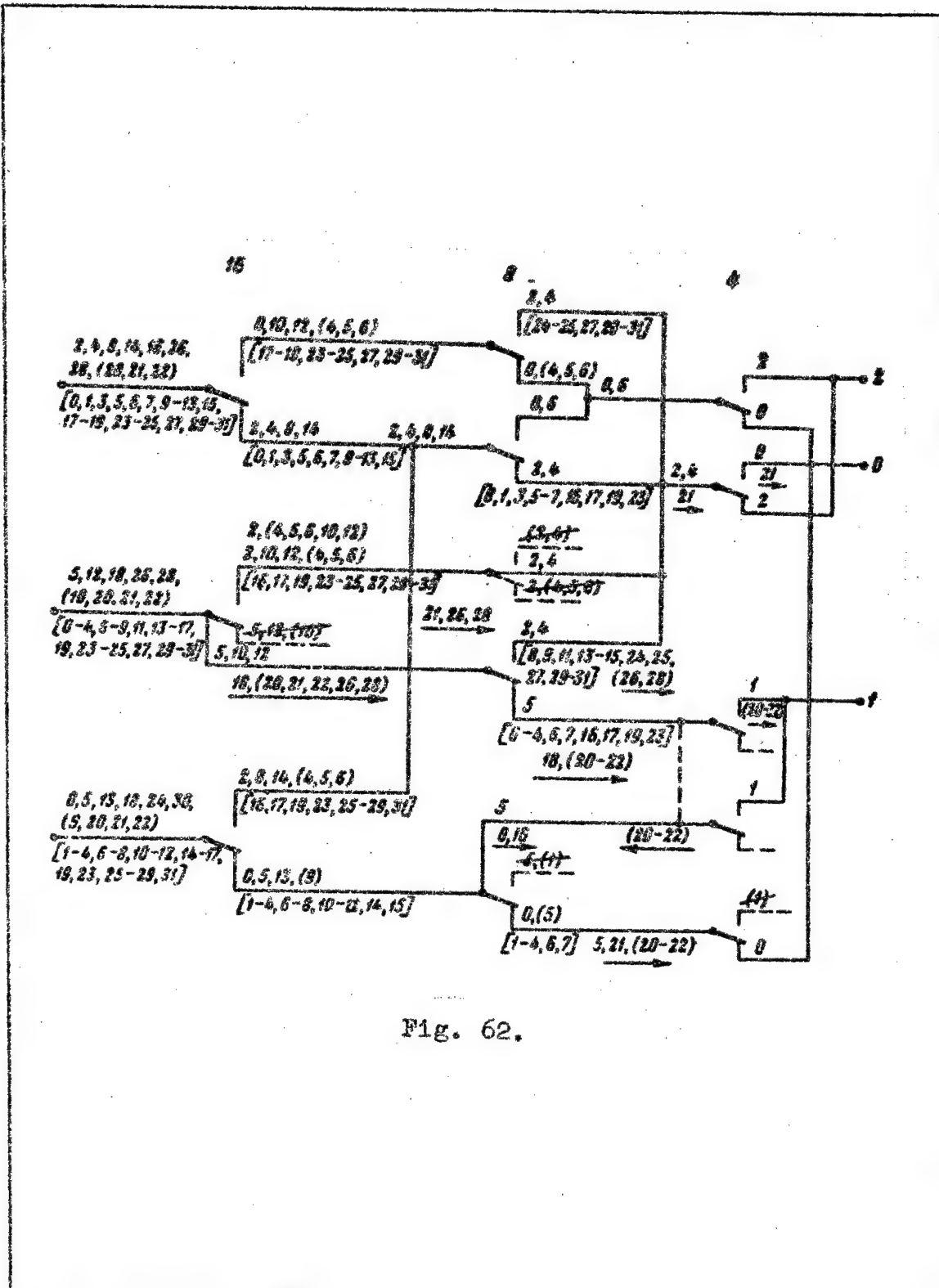
Fig. 62.

of the assembly f_{ji} goes into the number

$$a' = a \cdot 2^m + i.$$ (7.6)

And since each of the outputs is connected with the common input of the (1, k)-pole by circuits passing through each of the intermediate points i, then to each output we assign an assembly, which contains the transformed numbers of all the assemblies f_{ji} with identical indices j, so that

$$f_j = \sum_{i=0}^{p-1} (f_{ji} \cdot f_i).$$ (7.7)

We next construct the network in accordance with the method given above (Chapter 7, Section 3), starting with the relay having the greatest weight 2^{n+m-1}, until the contacts of the relay with weight 2^m are introduced into the network. Then the numbers around the free ends of the contacts of this relay will indicate the numbers of the inputs of the (p, k)-pole under construction.

By way of an example, we consider the synthesis of a (3, 3)-pole, made up of the contacts of three relays, specified by the assemblies listed in Table 16.

Since p = 3, m = 2. The transformed numbers are listed in the same Table 16. The resultant network is shown in Fig. 62.

Table 16

f_{ji}	Номера		
	исходные a	преобразованные $a' = 4a + i$	Набор выхода f_j
f_{00}	1,2,4,7,(5)	4,8,16,28,(20)	
f_{01}	(5)	(21)	2,4,8,14,16,26, 28,(20,21,22)
f_{02}	0,3,6,(5)	2,14,26,(22)	
f_{10}	3,7,(5)	12,28,(20)	
f_{11}	1,(5)	5,(21)	5,12,18,26,28, (10,20,21 22)
f_{12}	4,6,(2,5)	18,26,(10,22)	
f_{20}	0,6,(5)	0,24,(20)	
f_{21}	1,3,(2,5)	5,13,(9,21)	0,5,13,18,24,30, (9,20,21,22)
f_{22}	4,7,(5)	18,30,(22)	

1) Numbers, 2) initial, 3) transformed, 4) assembly of output j.

9. CHANGE OF ORDER OF CONSTRUCTION OF THE NETWORK

As already indicated in the graphical method of construction, the structure of the network may change, depending on the sequence with which the different contacts of the relays are introduced at another network, in analogy with how the complexity of an algebraic expression can vary with the order with which elements are taken outside the brackets. Depending on the sequence of the contacts of the relays, the constructed networks may differ in their structure and the number of springs, both total and per relay.

In the graphical method of network construction (Chapter 7, Section 3), the sequence of introducing the contacts depends on the

weights assigned to individual relays, i.e., the arrangement of the element in the base. Thus, in the general case, for a network consisting of n relays one can obtain n! different networks, corresponding to different permutations of the elements in the base. Therefore, to find the network satisfying certain requirements (for example, minimum contacts or springs), it may become necessary to construct all n! versions and to choose the best. There is still no general method that permits determining beforehand the base for which the simplest network is obtained.

However, it is not always necessary to construct all networks, for in some cases all the networks are obtained either quite identical, or of identical complexity.

It is known first of all /16/, that the so-called symmetrical contact networks do not change when the relays are renumbered. Consequently, if the network will be symmetrical, then no matter how we transpose the base, we always obtain the same assembly. The necessary and sufficient condition for an assembly to belong to a symmetrical network is that this assembly contain all the numbers of the states in which an identical number of relay operates, corresponding to the working numbers of the symmetrical network, and no other numbers.

The assemblies that characterize symmetrical networks with different working numbers are listed in Table 17 for n = 2, 3, 4, and 5. The assembly of a network with several working numbers

should include in their entirety the assemblies corresponding to each of the working numbers, and should not include any other numbers.

Thus, for $n = 4$, a symmetrical network with working numbers 2 and 3 will correspond to the assembly $\{3, 5, 6, 7, 9, 10, 11, 12, 14\}$, and for the inverse circuit (working numbers 0, 1, 4) the corresponding assembly will be $\{0, 1, 2, 4, 8, 15\}$.

Table 17 *

Рабочее число	n			
	2	3	4	5
0	0	0	0	0
1	1,2	1,2,4	1,2,4,8	1,2,4,8,16
2	3	3,5,6	3,5,6,9,10,12	3,5,6,9,10,12, 17,18,20,24
3		7	7,11,13,14	7,11,13,14,19, 21,22,25,26,28
4			15	15,23,27,29,30, 31
5				

1) Working number

* This table can be readily broadened by considering that the assembly for the working number p, in the case of n relays, is made up of the assembly for the same working number p for n - 1 relays, added to the assembly for the working number p - 1 for n - 1 relays, in which each number is increased by 2^n, i.e., $\{\alpha_n^p\} = \{\alpha_{n-1}^p, \alpha_{n-1}^{p-1} + 2^n\}$, where α_n^p is the assembly of a symmetrical network with a working number p for n relays.

Let us now imagine a network consisting of a series connection of a symmetrical network made of contacts of s relays -- A_1, A_2, ..., A_s --- and a non-symmetrical circuit of contacts of r relays -- B_1, B_2, ..., B_r. It is obvious that an interchange of the weights among the relays A, which make up the symmetrical circuit, will not change either the assembly of the entire circuit, or the network. On the other hand an interchange of the weights of relay B, as well as a mutual interchange of weights between relays B and A, will lead to a change in the assembly and may lead to a change in the network. In other words, such a circuit with a base including all the relays

substantially

A and B, may be, influenced only by those permutations, in which at least one relay B participates, while permutations of relays A do not influence the result.

permutation of

A network of the contacts of n relays, in which only part of the relays, s ($s <$ n), does not change the assembly, will be called partially symmetrical with respect to the relays A_1, A_2, ..., A_s.

For a partially symmetrical network, the total number of different assemblies is decreased by s!, i.e., the number will be n!/s!.

Distinguishing features of the partially-symmetrical network with respect to two relays with weights q_i and q_j ($i \neq$ j) will be the presence or absence in the assembly of the numbers q_i and q_j pairwise, as well as of other derivative numbers determined by

the states of the remaining relays. Thus, for example, in a network consisting of four relays, the assembly of a partially-symmetrical network relative to relays with weights 1 and 4 should contain or not contain the following pairs of numbers: 1 — 4, 3 — 6, 9 — 12, and 11 — 14. The presence of at least one of the unpaired numbers from among these will be evidence that the network is not symmetrical with respect to the relays with weights 1 and 4.

Table 18 lists the pairs of numbers which determine the symmetry with respect to two relays with weights indicated in the first column.

Table 18

$q_i - q_j$	n: 3		n: 4		n: 5			
1—2	5—6	9—10	13—14	17—18	21—22	25—26	29—30
1—4	3—6	9—12	11—14	17—20	19—22	25—28	27—30
2—4	3—5	10—12	11—13	18—20	19—21	26—28	27—29
1—8	3—10	5—12	7—14	17—24	19—26	21—28	23—30
2—8	3—9	6—12	7—13	18—24	19—25	22—28	23—29
4—8	5—9	6—10	7—11	20—24	21—25	22—26	23—27
1—16	3—18	5—20	7—22	9—24	11—26	13—28	15—30
2—16	3—17	6—20	7—21	10—24	11—25	14—28	15—29
4—16	5—17	6—18	7—19	12—24	13—25	14—26	15—27
8—16	9—17	10—18	11—19	12—20	13—21	14—22	15—23

It is easy to prove the following theorems.

1. If any circuit is simultaneously partially symmetrical with respect to relays with weights $q_i - q_j$ and $q_i - q_k$, then it

is partially symmetrical with respect to all three relays with weights q_i, q_j, and q_k.

2. If any circuit is simultaneously partially-symmetrical with respect to two groups of relays, in which there is at least one common relay, then the network will be partially symmetrical with respect to the aggregate of all relays of both groups.

By way of an example let us consider the network considered above (see p 96 /of source/), with circuits specified by the assemblies

$$X_0 = \{1, \underline{2}, \underline{4}, 7, 9, (6, \underline{11}, \underline{13}, 14, 15)\}_{\Gamma BBA};$$

$$X_1 = \{1, 7, 8, 9, \underline{10}, \underline{12}(0, 6, \underline{11}, \underline{13}, 14, 15)\}_{\Gamma BBA};$$

$$X_2 = \{\underline{2}, \underline{4}, \underline{3}, \underline{5}, (0, 6, \underline{11}, \underline{13}, 14, 15)\}_{\Gamma BSA}.$$

Using Table 18, we establish that all three circuits are partially-symmetrical with respect to relays with weights 2 and 4 (the corresponding pairs of numbers are underscored), i.e., relays B and C.

Thus, this circuit has $n!/s! = 4!/2! = 12$ different sets of assemblies, determined by the changes in the weights of the relays, and can have 12 different versions. Fig. 63 shows all these versions, constructed by the graphical method without the use of direct leads, while Fig. 64 shows the same networks with direct leads. For each network the base is indicated in the form of a fraction — the number of contacts in the network (numerator) and

Fig. 63

ГВБА
ГВАБ
ГАГБ

ВГБА
ВГАБ
АГВБ

ВБГА
ВАГБ
АВГБ

ВБАГ
ВАБГ
АВБГ
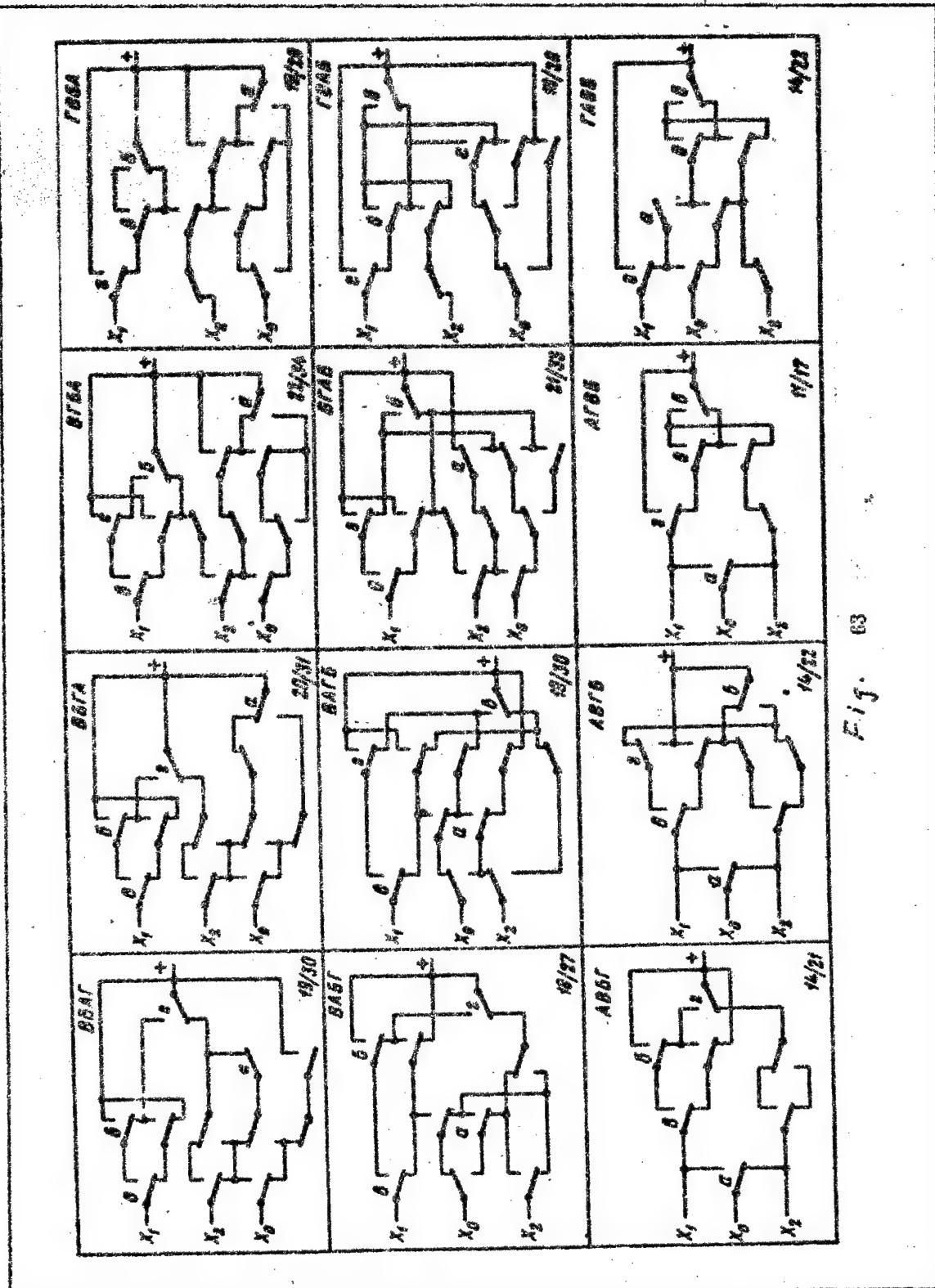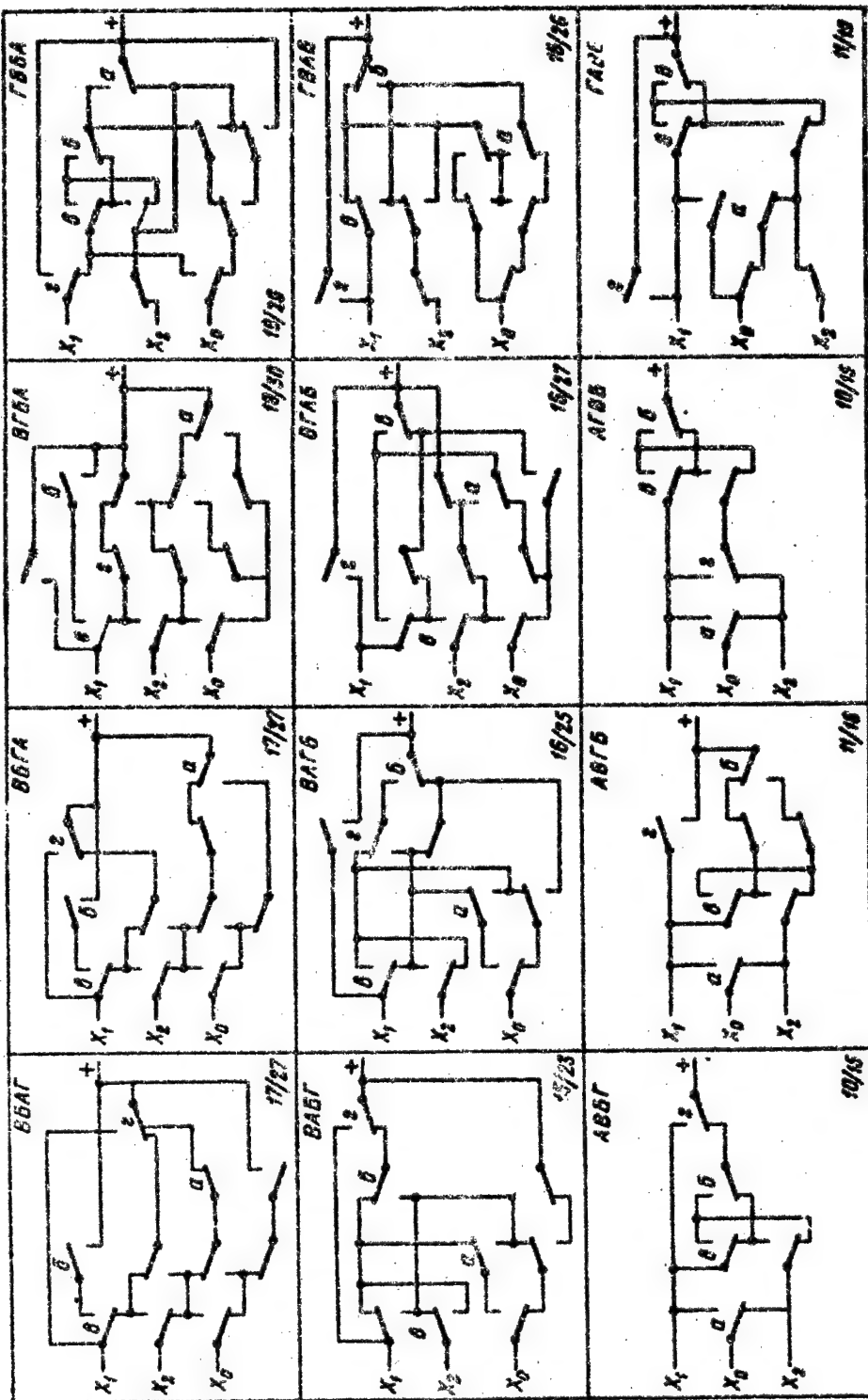
Fig. 64

the necessary number of springs (denominator). For comparison, Fig.
65 shows the network of the same circuits in the existing register
/122, pyramids II and III on network 25/, which have 27 contacts and
calls for 42 springs. The construction of the network in all cases
much simpler.

While in the case of a symmetrical network any transposition
of the base, or in the case of a partially-symmetrical network a
partial transposition of the base, does not change the assemblies in
the network, there may be cases when the transposition does change
the assemblies, but the constructed network does not become of
different complexity. The simplest example is a circuit with an
assembly containing only one number. For any placement of the con-
tacts, the network will always contain the same contacts, connected
in different sequences. The same pertains also to networks which
are of the same type in the sense of Polya /116/ with symmetrical
networks, i.e., obtained from symmetrical ones by inversion of
individual variables. At the present time there is no good method
for determination of such functions.

It is necessary to carry out additional investigations of
the properties of different networks, so as to make recommendations
in what cases it is necessary to investigate all the possible per-
mutations in order to find the optimal network, and in what cases
not all must be investigated, and also make recommendations on the
choice of the most suitable base.

To construct all possible networks one can change the weights of the relays in accordance with the change in the base or to carry out constructions by taking the contacts not in decreasing order of weights of the relay, but in an arbitrary order, transforming the assemblies as indicated in Chapter 6, Section 5. In the particular case when the contact group of the relay with weight 1 is chosen first, then all the even numbers are written at the make contact (starting with zero), divided by two, and the odd numbers, reduced by unity and also divided by two, are written at the break contact. The weights of the remaining relays are also divided by two. The construction is then continued in analogy with the previous construction, or starting with the relay having the highest of the newly-obtained weights, and the sequence indicated in Section 3 of the present chapter.

In the latter case it is not necessary to divide the numbers and weights of the relays by two after introducing the contacts of the relay with weight 1.

It must be noted that the graphic construction for a sequence different from that considered in Section 3 of the present chapter, leads to more complicated transformations of the numbers, and is particularly inconvenient in the case when it is necessary to verify the possibility of appearance of false circuits.

10. Construction of Inverse Circuits

To construct inverse circuits one can also propose a graphic method, which makes it possible to construct the network from the assembly of the main circuit without first finding the assembly of the inverse circuit. This method is based on the principle of graphic inversion of contact circuits. As a result of such an inversion, a $(1, k)$-pole of normal form (Fig. 66a) becomes the network shown in Fig. 66b, where the contact admittance between two neighboring points should be the inverse of the admittance between the corresponding output and the common input in the circuit of Fig. 66a.

To construct an inverse circuit with k outputs, we arrange along a horizontal $k + 1$ point, between which we write down the corresponding assemblies of the circuits. If there are coinciding assemblies, they should be arranged in a row, and then destroy the center point and write down the unified assembly.

Next, between each of the pairs of points we place the transfer contact of the relay with the largest weight $q_n = 2^{n-1}$, joining one of the points with the make contact and the other with the break contact. From the center springs of these contacts, as well as from the points corresponding to the outputs, we draw lines and make up new points, of which there will be $2k + 1$. Between each pair of new points we write down the assemblies in accordance with the following rules: between the points parallel
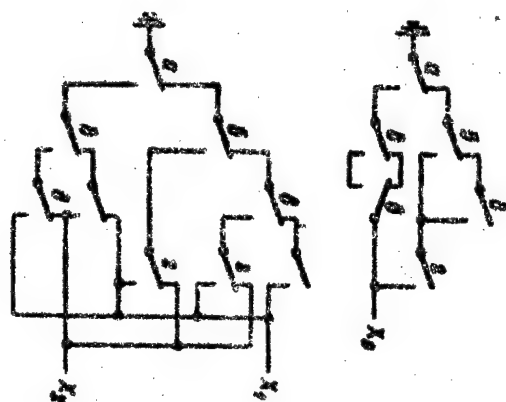
Fig. 65.
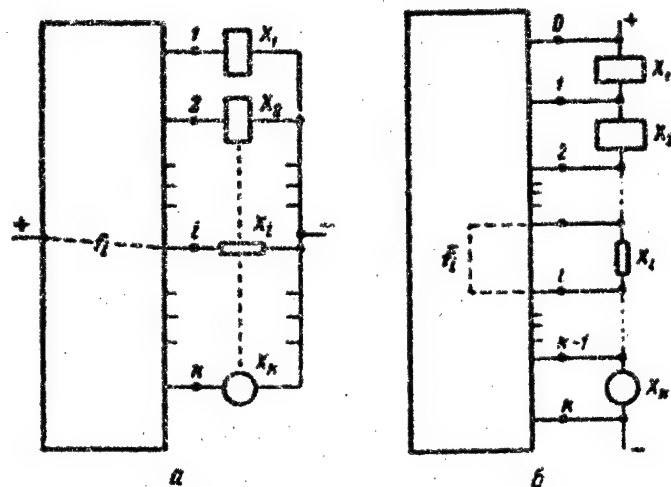
Fig. 66.

to which is connected the make contact of the relay with weight q_n we write out, from the assembly located above the contact, the numbers less than the weight q_n, while between the points parallel to which is connected the break contact we write out the remaining numbers, reduced by the weight of the relay.

We then make a comparison of all the assemblies in the following sequence:

1) We verify whether there are any coinciding assemblies, on both sides of the transfer contact, and if they exist, we erase the entire contact and the point joined to the center spring, and write down the unified assembly between the extreme points.

2) We see whether there are any empty assemblies and if such are found we join the points between which is located an empty assembly, and erase the corresponding contacts.

3) We verify whether there are any coinciding assemblies arranged in a row, and if they exist, we erase them   as well as the point between them, and write in this place the unified assembly. If the coinciding assemblies are located not in a row, then   we redraw the network in such a way that they appear in a row. In particular, one can interchange the places of the make and the break contacts.

Between the newly-obtained neighboring points, with the exception of the points between which is located the complete assembly, we draw the contact of the next relay with weight $q_n - 1$

and repeat the operation on the change and joining of the assemblies.

We continue this until the contacts of the relay with the lowest weight 1 are introduced into the network. We then redraw the network and unify where possible the individual contacts into transfer groups.

Fig. 67 shows by way of an example the construction of an inverse network for a binary divider of number of pulses.

We note that in the construction, the resultant network is sometimes planar, and sometimes, the complexity of such a network depends not only on the base used, but also on the sequence with which the points of the outputs are arranged and how the contacts are connected to the points, which is of no significance in the construction of normal networks.

Sometimes one can obtain in inverse networks simplifications analogous to the simplifications obtained in the construction of normal networks by using direct leads.

In the case when the two assemblies adjacent to the transfer spring of any transfer contact contain the same numbers, these assemblies can be transformed into three assemblies, in analogy with the separation of the assembly for the direct lead (see Section 4 of the present chapter). The distance between the points to which the transfer contact was connected, is then divided into three parts, in two of which we draw the make and the break contacts, and two points remain unconnected. Between them we

Fig. 67.

a

б

Fig. 68.

write down the separated assembly, and we assign to the contacts the transformed assemblies, as shown in Fig. 68a for the assembly given in Fig. 52.

In the particular case when one of the assemblies adjacent to the transfer contact is completely contained in an assembly of another contact, the contact corresponding to the first assembly can be erased, and in the second assembly, the numbers contained in the first assembly must be made conditional, as shown, for example, in Fig. 68b (compare with Fig. 53). With this aid of this rule one can eliminate from the network of Fig. 67 the contacts designated by crosses. We note that the joining of such a separated assembly with the neighboring   coinciding assembly can lead to disturbance in the operation of the network, analogous to what occurs when direct leads are joined.

11. APPLICATION OF THE GRAPHICAL METHOD FOR THE CONSTRUCTION OF CONTACT NETWORKS WITH MULTIPLE-POSITION TRANSFER SWITCHES

In the preceding sections we developed a graphical method for the construction of contact $(1, k)$-pole networks, made up of contacts of relays, i.e., contacts of two-position elements. We now extend this method to include the construction of networks made up of contacts of multiple-position elements, for example, step switches.

Let us consider networks of contacts of n switches A_1, A_2, ...

A_n, each of which has ℓ positions (blades). Since each switch can be in ℓ positions, the entire network will have ℓ^n different states.

In analogy with the relay-contact (two-position) networks, we number these states. For this purpose we assign to each switch A_i a weight $q_i = \ell^{i-1}$, i.e., for three-position switches we obtain weights 1, 3, 9, 27, for four-position switches we have 1, 4, 16, 64, etc.

We assign to each position of the switch the numbers $j = 0$, 1, ..., $\ell - 1$, and we assume the number of the state to be the sum of the products of the number of the position j_i in which each of the switches A_i is located in the given states, by the weight q_a of this switch, i.e.,

$$a = \sum_{i=1}^{n} j_i q_i.$$

(7.8)

Thus, the states will have numbers from zero to $\ell^n - 1$. Table 19 gives an example of the numbering of the states of the network made up of two three-position switches.

As in the case of a relay, any circuit of the contacts of the multiple-position switches can be written in the form of an assembly of numbers of those states, in which this circuit should or can be closed, i.e., in the form of an assembly of obligatory and conditional numbers.

The construction of the network proceeds in the same way as for relay contacts, the only difference being that every time one

draws not the transfer contact of the relay, but a switch with ℓ positions (blades), and the lead from each of these positions, number $j = 0, 1, \ldots, \ell - 1$, from the preceding assembly one assigns numbers from $j.q_i$ to $(j + 1)q_i - 1$, from which the number jq_i is subtracted.

Table 19

Переключатели A_i находятся в положениях с номерами 1)		Номер состояния схем α 2)	Состояния цепей 3)		
$A_2(q_2 = 3)$	$A_1(q_1 = 1)$		X_1	X_2	$X_.$
0	0	0	~	0	0
0	1	1	0	~	0
0	2	2	1	0	1
1	0	3	0	~	0
1	1	4	1	1	1
1	2	5	~	0	~
2	0	6	1	0	1
2	1	7	0	1	1
2	2	8	0	~	1

1) Switches A_i are in positions with numbers, 2) number of state of the network, α , 3) states of the circuits.

Thus, for example, for a three-position switch with weight $q_i = 3$ we find that to the zero lead $(j = 0)$ are given the numbers 0, 1, and 2, while to the first lead $(j = 1)$ — the numbers 3, 4, from which three is subtracted, and to the second lead $(j = 2)$ are given the numbers 6, 7, 8, from which six is subtracted.

By way of an example, let us show the construction (Fig. 69a) of a network for two three-position switches A_1 and A_2,
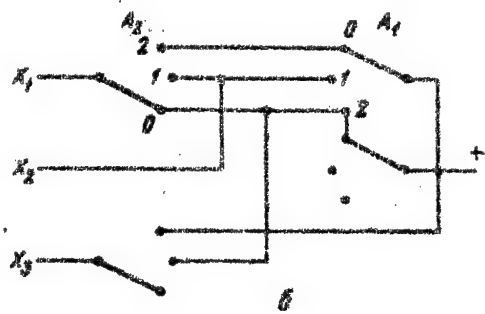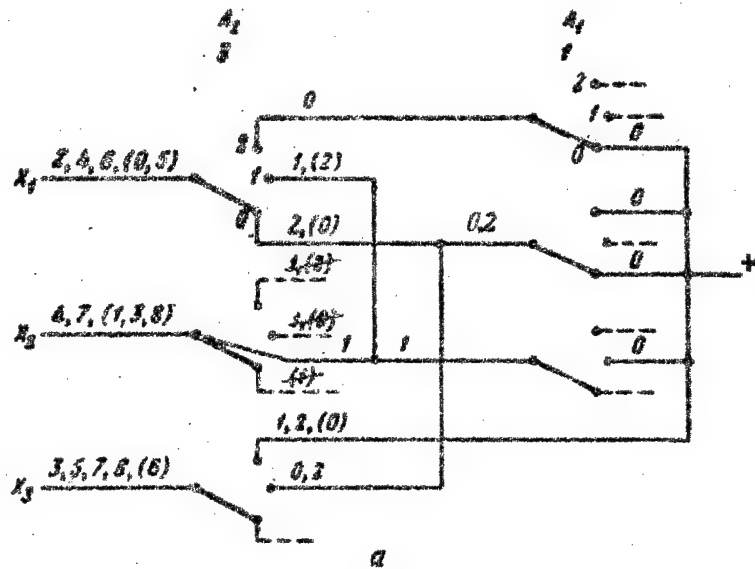
Fig. 69.

when the circuits are specified by the following assemblies (see table 19):

$$X_1 = \{2, 4, 6, (0, 5)\};$$
$$X_2 = \{4, 7, (1, 3, 8)\};$$
$$X_3 = \{3, 5, 7, 8, (6)\}.$$

Fig. 69b shows the same network in the redrawn form. As in the case of networks with two-position elements, in multiple-position switches the complexity of the network may depend on the sequence with which are assigned to the switches, and "direct" leads can be established during the construction.

Chapter 8

TRANSFORMATIONS OF RELAY NETWORKS

1. General Concepts of Relay Networks

In the preceding chapters we considered transformations of contact networks and circuits, and the main criterion for the equivalence was considered the equality of their structural admittances.

In relay networks, there are in addition to the contacts also the relay windings, as well as other elements such as resistances, rectifiers, capacitors, etc.

In the present chapter we shall consider relay networks, which may contain along with the relays only active resistances. In addition, we shall assume that the network is fed from a signal source, i.e., it has the form of a two-terminal network, the terminals of which are the points where the current source is connected.

In relay networks we are interested above all by the operating conditions of the relays contained in the network. The operation of a relay in a network, depends, on the one hand, on the state of the contacts, and on the other on the presence of active resistances in these circuits (including windings of other relays), and in the case of multiple-winding relays also on the interaction between

windings.

Therefore, before we proceed to the question of transformation of relay networks, we shall see on what the operation in a network depends and how this operation is influenced by the active resistances contained in the relay circuit.

2. Operating Conditions of a Relay in a Network

The state of a given electromagnetic non-polarized unblocked relay depends on the magnetic flux produced by the relay windings, and this can be characterized by the ampere turns (the product of the current flowing in the winding by the number of its turns).

The operation of a relay of a given construction and a given load is determined by the following values of the ampere turns, which characterize the limits of possible transition of the relay from one state into the other:

The non-operation ampere turns AW_n, i.e., the maximum value of ampere turns at which the relay will still not operate.

The operating ampere turns AW_o, i.e., the minimum number of ampere turns at which the relay will operate reliably.

The holding ampere turns AW_h, i.e., the minimum value at which the relay will continue to hold after it has operated.

The release ampere turns AW_r, i.e., the maximum number of ampere turns at which the previously-operated relay will be fully released.

The following relay is possible here

$$AW_o > AW_h > AW_{o \cdot r}$$

For most telephone relays we also have the relation

$$AW_n > AW_{o \cdot r}$$

The "working" ampere turns AW_W, i.e., the ampere turns which are produced in a given relay winding, depend on the parameters of this winding as well as on the voltage of the current source and other circuit parameters. If the following relation is satisfied

$$AW_p > AW_c \qquad\qquad (8.1a)$$

the relay will operate reliable, and if

$$AW_p < AW_o \qquad\qquad (8.1b)$$

it will be in the quiescent state.

For intermediate values of AW_W, i.e.,

$$AW_o < AW_p < AW_c.$$

the relay will remain in the state in which it was before the instant when those ampere turns were produced, or may be in an intermediate state.

Let us set in correspondence with each relay A (for simplicity of argument, we shall assume for the time being that it has only one turn) a coefficient λ_A, which will be taken to mean the ratio of the working ampere turns AW_W produced by the winding in any state of the network, to the operating ampere turns of this relay

$$\lambda_A = \frac{AW_{pA}}{AW_{cA}}. \qquad\qquad (8.2)$$

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(This coefficient is sometimes called the reliability margin in operation).

We shall assume this coefficient to be positive, if the current flows from the start of the winding to its end, and negative otherwise (we assume all relays to be wound in the same direction).

Both the quantity AW_w, and the number of turns are constant for a given relay, and therefore the coefficient λ_A will depend on the voltage of the power source and the state of the circuit, to which the given winding is connected. We shall henceforth, as is observed in most practical, that the voltage is constant.

Under these conditions the current in the relay winding, connected in the relay network, will depend on the resistances in the circuit of this relay, on the mutual connection, and on the connection with the current source. Since the configuration of the relay circuit depends on the states of the contacts, consequently the coefficient λ_A may change during the operation of the network and will be a discrete function of the states of the contacts of the network

$$\lambda_A = \lambda_A(a,\, b,\, \ldots,\, n). \tag{8.3}$$

The value of λ_A can change from zero (no current in the windings) to $\pm \Lambda_A$, the value of which is determined by the ampere turns produced by a given winding when the source voltage is

connected to its terminals, and the sign is determined by the direction of the current in the winding, i.e.,

$$0 \leqslant |\lambda_A| \leqslant \Lambda_A. \qquad (8.4)$$

For networks in which only pure contact circuits operate on the relay windings (without parametric action), λ_A can assume only two extreme values, depending on the state of the circuit f_A acting on this winding:

$$\lambda_A = \begin{cases} 0 & \text{for } f_A = 0 \\ \Lambda_A & \text{for } f_A = 1. \end{cases} \qquad (8.5)$$

If a finite conductance G is placed in series with the relay A (we shall use the concept of conductance rather than that of resistance for convenience in using the symbolic algebra of contact networks), as the conductance is increased from zero to infinity the coefficient λ_A will increase from zero to λ_A, as shown in Fig. 70a. To the contrary, in the case of parallel connection of the same conductance, the value of λ_A will decrease with increasing G (Fig.70c). (As in other cases in considering shunting action, we shall assume that the voltage-source circuit contains a suitable limiting resistance). In accordance with this, we can separate regions W of the values of the conductance G, at which a given relay in a given network operates reliably (shown shaded in Fig. 70b and d), and regions n, at which the relay never operates. Between these will lie the intermediate region, in which the state of the relay depends on its preceding state (or the relay can be an inter-

mediate state, some of the contacts have been switched over, and some not).

In accordance with (8.1), the region w of reliable operation of the relay A is determined by the relation

$$|\lambda_A| > 1 \qquad\qquad (8.6a)$$

and the region n by the relation

$$|\lambda_{\overline{A}}| \leqslant \frac{\Delta W_o}{\Delta W_c}. \qquad\qquad (8.6b)$$

For each state of the contacts of the relays of the network, if we know how the individual windings and resistances are interconnected and we know their parameters, we can determine the values of the coefficients λ, and from them judge the operation of the relay.

We now consider how the inclusion of active resistances in the circuit affect the operation of a relay.

3. ROLE OF ACTIVE CONDUCTANCE IN A RELAY CIRCUIT

As we have just seen, when an active conductance G is introduced in the winding circuit of relay A, the coefficient λ_A is reduced.

When a conductance G is added to the circuit of a single-winding A, depending on the value of this conductance, the relay can do the following:

a) Operate and hold ($\lambda_A > 1$).

b) Not operate, but hold, if it has operated previously
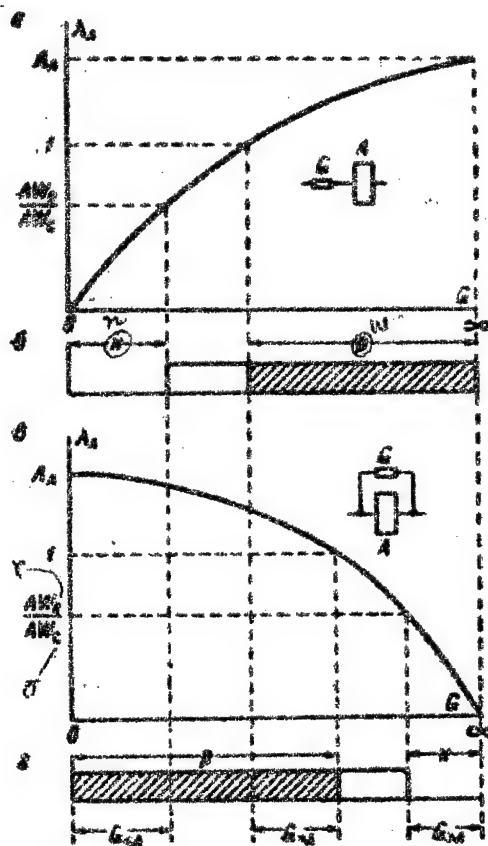
Fig. 70.

$\Gamma(1 > \lambda_A > AW_h/AW_o)$.

c) Neither operate nor hold $(\lambda_A \leqslant AW_r/AW_o)$.

In cases (a) and (c), the character of the effect of the connected conductance is independent of the state of the relay, whereas in case (b) these defects will differ depending on whether the relay has previously operated or not.

Therefore in the analysis of the operation of relay A the conductance G can be replaced by the value indicated in Table 20.

Table 20

1) Действие реле А при подключении к нему G	2) Если проводимость G подключена к А, то она заменяется на:	
	3) при последовательном включении	4) при параллельном включении
5) Реле срабатывает и удерживает	1	0
6) Реле не срабатывает, но удерживает	a	a
7) Реле не срабатывает и не удерживает	0	1

1) Action of relay A when G is connected to it.

2) If the conductance G is connected to A, it is changed by:

3) In series connection

4) In parallel connection

5) Relay operates and holds

6) Relay does not operate but holds

7) Relay operates and does not hold

If there are several conductances, the operating conditions of

the relay can be more complicated. Thus, for example, the same conductance, connected in parallel to a relay, depending on the presence or absence of another conductance connected in series, may either disturb or not disturb the operation of the relay.

The dependence of the operation of a relay connected in the diagonal of a bridge made up of active conductance is particularly complicated.

As yet there is no general method for converting relay networks with active conductances. However, for the case of class π networks, under certain limitations, which will be discussed later, one can give an analytical method of equivalent transformations of relay networks, which can be used above all in their synthesis.

4. CONCEPT OF THE ORDER OF THE CONDUCTANCE

In order to characterize the effect of a finite conductance G connected to a relay A, we introduce the concept of the order of the conductance. Each conductance G will be judged in accordance with how it effects the operation of a given relay A in a given network in series and in parallel connection. With this, we shall consider only the regions in which reliable operation or release of the relay is insured (regions w and n in Fig. 70).

If the conductance G, connected to relay A, both in series and in parallel, does not disturb the operation of this

relay, we shall say that this conductance is of one order with relay A, and we shall agree to designate it by $G_{=A}$.

If, however, the addition of the conductance G in series with the winding of the relay A disturbs completely the operation of the relay, while parallel connection of the same conductance does not disturb the operation, we shall say that the conductance G has a lower order with respect to the relay A and will be denoted by $G_{<A}$.

To the contrary, if parallel connection of the conductance disturbs the operation, while series connection does not affect it, we shall say that the conductance G is of higher order with respect to relay A, and we shall denote it by symbol $G_{>A}$.

Finally, a case may be when the operation of the relay A is disturbed both by parallel and by series connection of the conductance G. Such a case, however, has no practical significance and we shall not consider it.

We note that the concept of the "order of conductance" is relative, since it characterizes not the absolute value of the conductance (or resistance), but the effect of this of this conductance on the operation of the relay in the network, i.e., it pertains to the specific parameters of the network and to the voltage applied to it.

It is seen from the definitions that withoutdisturbing the operation of the relay A, one can connect in series txxt with the

winding of this relay a conductance of the same order or higher (we shall denote it in general form by $G_{\geqslant A}$). On the other hand, a conductance $G_{< A}$, connected in series, causes   disturbance of the operation of the relay A. Analogously, the parallel connection of a conductance of lower order or of the same order (denoted in general form $G_{\leqslant A}$) will not affect the operation of the relay, while a parallel-connected conductance of higher order $G_{> A}$ shunts the relay and disturbs its operation.

We shall agree that if the formula or a certain separated part of the formula does not have any indication of the order of the conductances, contained in the formula for elements of finite conductance (in particular, relay windings), then all these elements have the same order of conductance.

5. INTERACTION OF RELAYS IN A NETWORK

In the presence of several relays in a network,   each relay represents a finite conductance with respect to the windings of the other relays, and they can influence the operation of each other. Without considering all possible cases of mutual influence of the relay windings connected in the network, we shall confine ourselves to the case (which is most important from the point of view of practice) when the parameters of the relays are so chosen, that the following conditions are satisfied:

$$
\begin{array}{lll}
\text{а) если } A_{-Б}, & \text{то } Б_{-A}; \\
\text{б) если } A_{>Б}, & \text{то } Б_{<A}; \\
\text{в) если } A_{<Б}, & \text{то } Б_{>A}.
\end{array} \right\} \qquad (8.7)
$$

From the definitions and limitations assumed above, we can obtain the following properties:

1) Transitivity:

$$\text{If } A_{>B} \text{ and } B_{>C}, \text{ then } A_{>C} \text{ or } C_{<A}; \qquad (8.8)$$

2) equality of orders:

$$\text{If } A_{\geqslant B} \text{ and } B_{\geqslant A} \text{ then } A_{=B} \text{ or } B_{=A}; \qquad (8.9)$$

3) non-realizability:

$$\text{If } A_{>B} \text{ and } B_{<A}, \text{ then the network is not realizable.} \quad (8.10)$$

It follows from the definitions that if, for example, three relays -- A, B, and C -- satisfying the relation $A_{>B}$ and $B_{>C}$ (or $B_{<A}$ and $C_{<B}$) are connected in series and to the power supply, only relay B, with the lowest order of conductance, will operate. In the case of a parallel connection of these relays, only relay A with the largest order of conductance will operate.

If all three relays have the same order of conductance, all will operate simultaneously both in the case of series and in the case of parallel connections.

Let us agree also, that the values of the conductances and the parameters of the relays are so chosen that in parallel or series connection of a finite number of conductances of one order, the total conductance will be of the same order, i.e.,

$$A + Б_{-A} + \ldots + K_{-A} = (A + Б + \ldots + K)_{-A};$$
$$A \cdot Б_{-A} \ldots K_{-A} = (A \cdot Б \ldots K)_{-A}.$$

$$(3.11)$$

For conductances of different order, the total conductance in

series connection will be determined by the element having the

conductance of lowest order, and in parallel connection by the ele-

-order

ment with the highest conductance, i.e.,

$$Б_{>A} + B_{<A} = Б_{>A};$$
$$Б_{>A} \cdot B_{<A} = B_{<A}.$$

$$(3.12)$$

From this it follows, in particular, that

$$0 \cdot G = 0; \qquad 1 \cdot G = G;$$
$$0 + G = G; \qquad 1 + G = 1.$$

$$(3.13)$$

6. Equivalence of Relay Circuits

As already indicated, the principal factor in relay networks

is the operation of the relays. Relay networks   which have

different structures but signals of the same character and sequence

applied to the elements lead to identical states of the actuating

circuits and to identical operation of the actuating elements, will

be called equivalent.

In the literature /16/ relays are sometimes called "equal".

Care should be taken to distinguish between

equivalence of contact networks as regards to their structural

admittance, and equivalence as regards the operation of the relay

in the network.

In the present book we shall consider not all possible equivalent networks, but only equivalent networks with the same number of relays and the same operating sequence, i.e., networks the action of which is determined by one and the same table of connections. In other words, we consider here transformations of relay networks, for which the operating functions of each relay and of each actuating circuit are specified.

If in addition to that, the networks have the same conductance, accurate to   within its order, we shall call them absolutely equivalent. Knowledge of the conductance of a relay network is important from the point of view of the mutual   influence of individual relay networks when they are connected into a common network.

We shall distinguish the following transformations of relay networks:

1) Equivalent, which lead to networks with the same operation. We shall introduce for these transformations the symbol $\rightleftharpoons$.

2) Equal, which lead to networks of the same (accurate to within its order) structural admittance. For these transformations, in analogy with contact circuits, we shall use the   equal sign, $=$.

3) Absolutely equivalent, which do not change the operation of the network and its structural admittance. For these trans-

formations we shall use the symbol $\equiv$.

7. Equivalent Transformations of Relay Networks

We have seen that   in the case of contact circuits, equal
transformations of the algebra of the contact networks lead to con-
tact networks which have the same structural admittance, i.e., net-
works are obtained, which will be simultaneously closed or opened
for different states of the relays, whose contacts enter into these
networks. However, the formalism of the algebra of contact networks
cannot always be used for transformations of relay networks, since
we are interested not in the structural admittance of the relay net-
works, but in the relay operation. In addition, the laws of the
algebra of contact networks are not always valid for elements of
finite conductivity (in particular,   inversion
is not defined for these elements). A relay circuit containing
elements of finite conductivity (in the sense of its influence on
the operation of the relay) can be considered as having values zero
or one, and therefore, with certain restrictions, the laws of the
algebra of contact networks can be used also for the transformation
of relay networks.

Let us see now what transformations can be carried out with
relay networks in order to obtain equivalent networks with allow-
ance for everything said above.

It is easy to verify that one can apply to networks of class
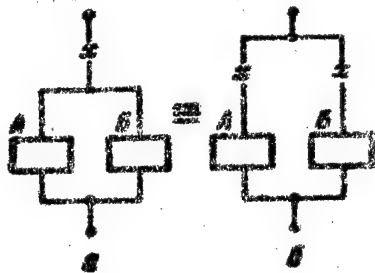
Fig. 71.
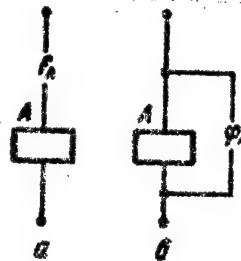
Fig. 72.
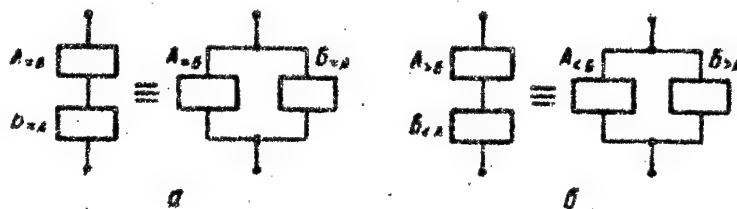
Fig. 73.
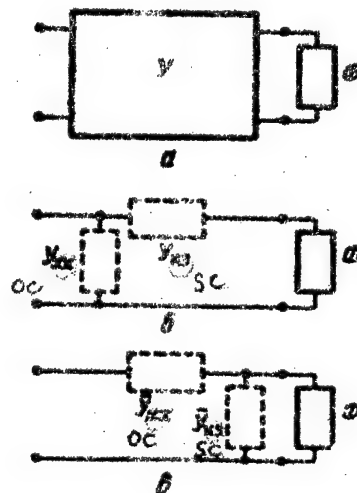
Fig. 74.

π, containing both contacts and windings of relays, those laws of ~~algebraic~~ the algebra of contact networks, which do not lead to a change in the number of windings. Above all, one can apply to relay networks, without any limitation whatever, the commutitive and associative laws of the algebra of contact networks, i.e.,

$$xA \equiv Ax; \\ x + A \equiv A + x; \Big\}$$ (8.14)

$$xyA \equiv (xy)A \equiv x(yA); \\ x + y + A \equiv (x + y) + A \equiv x + (y + A). \Big\}$$ (8.15)

The distributive laws of the algebra of contact networks are applicable only for those cases, when the number of windings does not change, i.e., for the interaction of the contact circuit relative to addition and multiplication of elements of finite conductance (Fig. 71):

$$x(A + Б) \equiv xA + xБ; \\ x + AБ \equiv (x + A)(x + Б). \Big\}$$ (8.16)

In formulas (8.14) -- (8.16), x and y can mean not only contacts of individual relays, but contact circuits, while A and B can mean not only the windings of individual relays, but relay circuits.

In addition, it is easy to verify that if the network consists of windings of single-winding relays with conductance of the same order and if the sufficient condition for the operation of a relay is that current flow through the winding of this relay, than

all networks of class π will be equivalent, since for any connections between these windings the current will pass through each of them, and all relays will operate simultaneously. In other words, for elements of finite conductivity of the same order we can write the following absolutely equivalent transformation.

$$A_{-B} + Б_{-A} \equiv A_{-B} \cdot Б_{-A}. \qquad (8.17)$$

If the windings of the relay have conductivities of different order, then the method of joining the windings will play an important role, due to the limitation of the currents in the windings of the individual relays, as indicated above, and consequently transformation (8.17) is not applicable in this case.

8. Inversion of Relay Networks

In the analysis of transformations of contact networks we defined inversion as finding a network, of inverse admittance. In the inversion of any circuit of class π, parallel connections become series connections and vice versa, while make contacts become break contacts and vice versa.

Let us examine now what happens in a relay network if inversion is carried out in the aforementioned sense.

Upon going from a series connection of a contact circuit and a relay winding (Fig. 72a) to a parallel connection (Fig. 72b) the operation of relay A will not be disturbed only if the circuit ϕ_A, shunting the relay A, will have an admittance inverse of

the circuit f_A, i.e., if these circuits are mutually inverse

$$\varphi_A = \overline{f}_A. \qquad\qquad (8.18)$$

We see thus that by inverting a relay circuit in accordance with the rule of inversion of contact circuits, but without changing the windings of the relay, we obtain a new network, operating in the same manner. It is easy to verify, however, that the two networks differ in their conductivity.

Analogously, in going from series connection to parallel connection for a circuit containing relays of one order of conductivity (Fig. 73a), the operation of the circuit does not change if the conductivities remain of the same order.

In the presence of windings of different orders of conductivity, upon going from series connection to parallel connection, the operation of the circuit will not change only if the orders of the

conductances of the relay windings in the network will be reversed (Fig. 73b). If we consider the conductances of the network, we shall find that under the foregoing conditions, in the case of inversion of a network consisting of conductances of the same order, the conductance of the network remains of the same order, but if the orders are different the order is reversed.

We can derive from this the following rules for inversion of elements of finite conductivity.

In networks with elements of the same order, inversion of

the network does not change their order, and if conductances of different orders are present in the network, inversion causes their order to be   changed from larger to smaller, and vice versa, i.e.,:

$$\overline{G_{=A}} \equiv G_{=A};$$
$$\overline{G_{>A}} \equiv G_{<A}; \quad \overline{G_{>A}} \equiv G_{<A};$$
$$\overline{G_{<A}} \equiv G_{>A}; \quad \overline{G_{<A}} \equiv G_{>A}.$$

$$(8.19)$$

Thus, the laws of inversion as applied to relay networks will have the form

$$\overline{x A} \equiv \bar{x} + A; \qquad \overline{x + A} \equiv \bar{x} \cdot A;$$
$$\overline{x A_{>B}} \equiv \bar{x} + A_{<B}; \quad \overline{x + A_{>B}} \equiv \bar{x} \cdot A_{<B};$$
$$\overline{x A_{>B}} \equiv \bar{x} + A_{<B}; \quad \overline{x + A_{>B}} \equiv \bar{x} \cdot A_{<B}.$$

$$(8.20)$$

Under these conditions the inversion of a relay network is an equivalent transformation, since the active of the network does not change. In other words, we can write

$$\overline{F} \doteq F.$$

$$(8.21)$$

The structural admittance of the network, however, is inverted when the network is inverted.

Along with this, one can broaden the process of inversion in relay networks and invert not the entire relay network, but only a part of it. (Individual procedures for partial inversion of relay networks are given in reference /131/.) Thus, if a relay two-terminal network $\emptyset$ is isolated in a relay network, and if this two pole network includes all the relay windings of the network

then one can invert either only the two-terminal network $\emptyset$, or the entire network, considering the two-pole $\emptyset$ unchanged.

To prove this statement, let us represent such a relay network in the form of a contact four-terminal network Y and a relay two-terminal network $\emptyset$ (Fig. 74a). If we consider $\emptyset$ as the load of the relay network, then from the conditions for separation of the circuits acting on this load /16, 18/, the contact four-pole can be represented in the form of a L-shaped link (Fig. 73b), in which the longitudinal arm contains a circuit with admittance equal to the input admittance Y_{sc} of the four-terminal network in which the output terminals are short circuited ($\emptyset = 1$), and in the shunt arm is the input admittance Y_{oc} with the output open circuited ($\emptyset = 0$).

In this case there will be voltage on the load if and only if the series branch is closed ($Y_{sc} = 1$) and the shunt branch is opened ($Y_{oc} = 0$), i.e., when

$$Y_{sc} \cdot \overline{Y}_{oc} = 1.$$

If we now invert the network, leaving $\emptyset$ unchanged, we obtain a new network (Fig. 74c) with a series branch $\overline{Y}_{oc}$ and a parallel branch $\overline{Y}_{sc}$. For this circuit, the condition for the presence of voltage on the load will be the same: the series branch closed ($\overline{Y}_{oc} = 1$) and the shunt branch opened ($\overline{Y}_{sc} = 0$):

$$\overline{Y}_{x.x} \cdot \overline{(\overline{Y}_{k.s})} = \overline{Y}_{x.x} \cdot Y_{k.s} = 1,$$

i.e., the conditions remain the same.

It follows therefore that upon inversion of a network in

which the two-pole $\emptyset$ remains unchanged, the conditions for the flow of current through the latter will be the same as prior to inversion.

However, in this case the input admittance (as in other cases of inversion) is changed. Actually, if the input admittance of the network prior to inversion was

$$Y_1 = Y_{x.x} + Y_{x.s} \cdot \Phi,$$
$$\text{oc}$$

it becomes after the inversion

$$Y_2 = \overline{Y}_{x.x}(Y_{x.s} + \Phi).$$
$$\text{sc}$$

The two-pole $\emptyset$ can in turn be inverted in accordance with the rule of inversion of relay networks. Actually, as we know, inversion of a relay network does not change its operation, but only its admittance. Inasmuch as the four-terminal Y is a purely contact network, a change of the order of the admittance of the load does not change the condition of current flow through it.

By way of an example let us consider the network of the form $F = xyA$, the inversion of which leads to the network

$$F = \overline{x} + \overline{y} + A.$$

Using the commutation rule and separating in different manners the contact two-pole (for the sake of clarity the separated part is under- lined), we obtain

$$F = x\underline{y}A \div x(\overline{y} + A) \div \overline{x} + yA$$

or

$$F = y\underline{x}A \div y(\overline{x} + A) \div \overline{y} + xA.$$

All the networks obtained (Fig. 75) are equivalent, but have
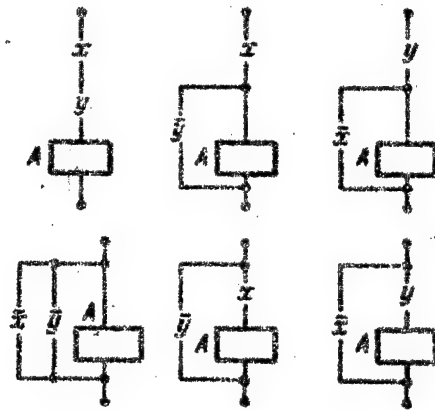
Fig. 75.
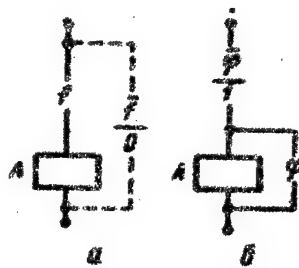
Fig. 76.

different structural admittances.

9. EXPANSION OF RELAY NETWORKS BY INTRODUCING CONTACT CIRCUITS

A relay network can also be modified by introducing in it, in accordance with definite rules, contact circuits for elements of finite conductivity. We shall now consider under what conditions one can introduce new contact circuits into a relay network.

It is quite obvious that the "expansion" of individual contact circuits in accordance with the laws given in Chapter 4 does not change the operating principle of the relay network, since under these expansions the admittance of the contact circuits does not change.

The introduction of contact circuits in relay networks offers greater possibilities of expanding the networks. Thus, if in a normal network of the form fA we   add in parallel a purely-contact circuit, the condition of operation of the relay A remains unchanged only if the circuit ψ is opened at the instant when the circuit f is closed. In other words, the following conditions should be satisfied.

$$f\psi = 0.$$

It is obvious that this condition is satisfied above all by an expression inverse to f, ~~imaxx~~ and also by any "smaller" expression, i.e.,

$$\psi < \bar{f}.$$

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$$\psi = \frac{\bar{I}}{0}.$$

We can thus write down the following equivalence (Fig. 76a):

$$|A \rightleftharpoons |A + \frac{\bar{I}}{0}. \qquad (8.22)$$

Analogously, the equivalence will not be violated for an inverse circuit if we connect in series a circuit which will be closed when the circuit connected parallel to the relay winding is opened. This corresponds to multiplying an expression of the form $\phi + A$ by an expression which is inverse to ϕ, or by some "larger" expression, i.e., (Fig. 76b)

$$\varphi + A \rightleftharpoons (\varphi + A)\frac{\bar{\varphi}}{1}. \qquad (8.23)$$

It is easy to verify that in transformations (8.22) and (8.23) the overall admittance of the circuit changes (with the exception of trivial cases $\frac{\bar{f}}{0} = 0$ and $\frac{\varphi}{1} = 1$), and consequently, these transformations are not absolutely equivalent.

10. EXPANSION OF RELAY NETWORKS BY INTRODUCING ELEMENTS OF FINITE CONDUCTIVITY

Let us consider the possibility of expanding a network by introducing elements of finite conductivity. In accordance with the definition, by connecting a conductance $G_{\geq A}$ in series with the winding of relay A, of the same order or greater, does not change the operating conditions of this relay. Naturally, the operating conditions will remain unchanged if the conductance of

this circuit is further increased by connecting in parallel to $G_{\geqslant A}$ some circuit Ω (purely-contact or with element of finite conductivity) [but not with relay windings . Elements of finite conductivity can be replaced by windings if circuits of several relays are combined (see p 138).] It follows therefore that one can connect in series with any relay network, without disturbing its operation, a circuit of the form $G_{\geqslant A_m} + \Omega$, where A_m is the network relay winding with the largest order of conductance (Fig. 77a):

$$F(a_1, \ldots, a_n, x, \ldots, z, A_1, \ldots, A_n) \doteqdot$$
$$\doteqdot (G_{> A_m} + \Omega) F(a_1, \ldots, a_n, x, \ldots, z, A_1, \ldots, A_n) \qquad (8.24)$$

Analogously, we can connect in parallel to each relay network a circuit of the form $\Omega G_{\leqslant A_\ell}$ (where A_ℓ is the network relay winding, having the smallest order of conductance):

$$F(a_1, \ldots, a_n, x, \ldots, z, A_1, \ldots, A_n) \doteqdot \Omega G_{< A_\ell} +$$
$$+ F(a_1, \ldots, a_n, x, \ldots, z, A_1, \ldots, A_n). \qquad (8.25)$$

These equivalences are in general not absolute. They become absolute only when the structural admittances of F and Ω are related as follows:

If $F = 1$, then $\Omega = 1$ for the equivalence (8.24) (8.26)

If $F = 0$, then $\Omega = 0$ for the equivalence (8.25).

In the case when the network F is of the normal or inverse type, i.e., when its structural admittance cannot assume the values of conditions (8.26), then the equivalences (8.24) and (8.25) becomes absolute (Fig. 77b and c):
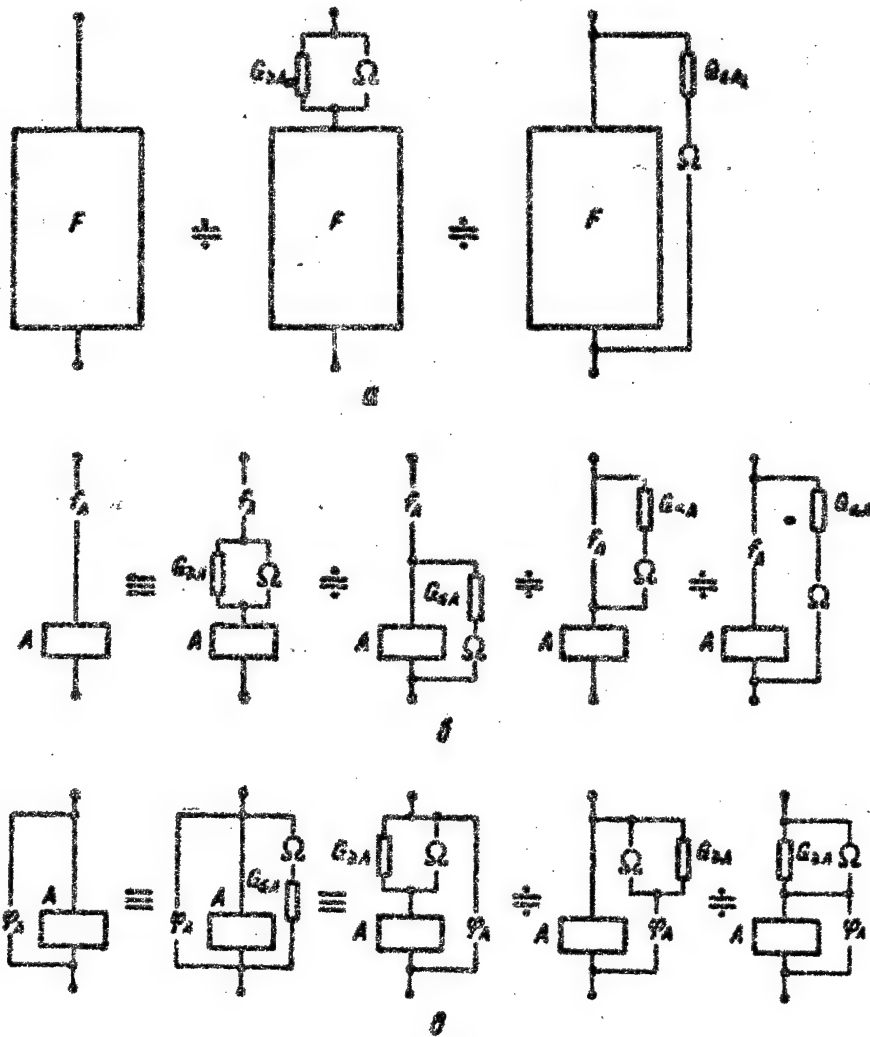
Fig. 77.

$$f \cdot A \equiv (G_{>A} + \Omega \cdot) f \cdot A; \qquad (8.27)$$
$$\varphi + A \equiv \Omega G_{<A} + \varphi + A. \qquad (8.28)$$

The circuit $G_{>A} + \Omega$ can also be introduced in series, and the circuit $\Omega G_{<A}$ in parallel with the winding of the relay A, i.e., one can add to formulas (8.27) and (8.28) the following equivalences:

$$f A \equiv f (A + \Omega G_{<A}); \qquad (8.29)$$
$$\varphi + A \equiv \varphi + (G_{>A} + \Omega) A. \qquad (8.30)$$

Elements of finite conductivity can also be introduced into individual circuits, acting on the relay A. Thus, an element with a conductivity $G_{>A}$ or a circuit of the form $G_{>A} + \Omega$ can be connected in series with any contact circuit, connected in series with the winding of the relay A.

In other words, in any term contained in f one can introduce as a multiplying factor the expression $G_{>A} + \Omega$ (in the particular case $\Omega = 0$) without disturbing the operation of the network:

$$(f_1 + f_2) A \equiv [f_1 (G_{>A} + \Omega) + f_2] A \equiv [f_1 + f_2 (G_{>A} + \Omega)] A. \qquad (8.31)$$

On the other hand, the operation of the network remains undisturbed if we connect ~~in parallel with the contact circuit~~ a conductance of the form $\Omega\, G_{<A}$, whose order is less than the conductance of the relay winding, in parallel to the contact circuit connected in series with the winding of relay A, since by definition in cases when the contact circuit is open the current will not be sufficient in the relay winding to operate this relay.

In other words, one can add to the formula of the contact circuit of a normal network of relay A an expression $\Omega G_{<A}$ as an additive term to any part of this circuit (in particular case $\Omega = 1$):

$$fA \doteq (f + \Omega G_{<A})\, A; \qquad (8.32a)$$
$$f_1 f_2 A \doteq (f_1 + \Omega G_{<A})\, f_2 A. \qquad (8.32b)$$

Analogously, in contact circuits connected in parallel to the winding of relay A, one can add in series a conductance $G_{>A} + \Omega$, while in parallel one can add a conductance $\Omega G_{\leq A}$:

$$\varphi_1 \varphi_2 + A \equiv \varphi_1 (\varphi_2 + \Omega G_{<A}) + A; \qquad (8.33)$$
$$\varphi + A \doteq \varphi (G_{>A} + \Omega) + A; \qquad (8.34a)$$
$$\varphi_1 + \varphi_2 + A \doteq \varphi_1 (G_{>A} + \Omega) + \varphi_2 + A. \qquad (8.34b)$$

If formulas (8.27) -- (8.34) are compared with (4.23a) and (4.23b) for the expansion of contact networks, as well as with formulas (8.22) and (8.33), an interesting analogy can be made between the concept "order of conductance" and the concept of larger or smaller contact circuits. Actually, where a larger circuit can be connected to a contact network without affecting the overall admittance,   one can introduce in a relay circuit an element of finite conductivity of greater order without disturbing the operation of the network. An analogous correspondence exists between the smaller contact circuit and a lower-order element of   finite conductivity.

11. ACTION OF MULTIPLE-WINDING RELAYS

In the cases considered above it was assumed that each relay has a single winding (one responding organ). In practice, one uses extensively multiple-winding relays, the action of which depends not only on the state of the circuits in which individual are connected, but also on the interaction of these windings.

For a multiple-winding relay we number (in the general case the numbering order is immaterial) all the windings of the relay and write down the number of the winding as a superscript to the symbol of the relay, i.e., A^1, A^2, etc. denote the first, second, etc. windings of the relay A. In addition, we agree that if in winding A the current flows from the end to the beginning, the symbol for this winding will be $\overleftarrow{A}^k$.

We note that it is not always essential to write down the winding numbers. This pertains in particular to intermediate transformations, in the case when the windings of one relay are all of the same order of conductance.

In a multiple-winding relay the magnetic flux is proportional to the total ampere turns produced by all the windings, with allowance for the direction of the current in the individual windings. If we denote by λ_{A^i} the ratio AW_o^i/AW_o, where AW_w^i are the "working" ampere turns produced by winding A^i of relay A, then the operation of relay A can be characterized by the algebraic sum of these coefficients for all the windings of this relay:

$$\lambda_A = \sum_i \lambda_A^i.$$

(8.35)

There are still no general methods for transforming relay networks with multiple-winding relays, and in the next sections we shall only discuss several particular cases, which are of practical significance. We note above all that the transformation of relay networks will depend on whether all the windings are connected in a coordinated manner (the current flows in all windings in only one direction), or whether there exists bucking windings.

12. RELAYS WITH COORDINATED WINDINGS

For relays with coordinated windings, we introduce the following limitations:

For operation of the relay it is sufficient that the circuit be closed of at least one of the windings and that the relations that follow from the concept of the order of conductance be retained.

When conductances of lower order are connected in series with the windings or conductances of higher order are connected in parallel, the relay will not operate or hold, even though current flows simultaneously in all windings.

In other words, we shall assume that the parameters of the network are chosen such that in the case when the relay should operate the coefficient $\lambda'_{A}i$ for any of its windings will be not less than unity

$$\lambda'_{Ai} \geqslant 1. \tag{8.36a}$$

and in those periods when the relay should not operate, the coefficients λ''_{Ai} are such that the following relation holds

$$\sum_{i} \lambda''_{Ai} < \frac{AW_o}{AW}. \tag{8.36b}$$

Under these limitations, the repetition law will hold, i.e.,

$$\left. \begin{array}{l} A \doteq A^1 + A^2 + A^3 + \dots \\ A \doteq A^1 \cdot A^2 \cdot A^3 \dots, \end{array} \right\} \tag{8.37}$$

as well as the distributive laws

$$\left. \begin{array}{l} (x+y)\,A \doteq xA^1 + yA^2, \\ xy + A \doteq (x + A^1)(y + A^2). \end{array} \right\}$$

With the aid of these relations one can introduce into the network additional windings, which in some cases makes it possible to simplify the network.

We note incidentally that the distribute laws, from the point of view of operation of the relay A, do not impose any limitations whatever on the sequence of the conductances of the windings A, A^1, and A^2. If it is assumed, however, that all the windings have the same order of conductance, then formulas (8.37) and (8.38) yield absolutely equivalent transformations.

We note furthermore that the formulas for the distributive laws will hold also in the case when x and y will contain elements of finite conductivity, but the formulas will not give

equivalent transformations if x and y will contain windings of
any relays.

The introduction of supplementary windings with use of their
laws (8.38) makes it possible to subdivide the contact circuit into
several independent circuits, a fact which can be used to reduce the
number of contacts when constructing multiple-relay networks, as
will be shown later.

After introducing into the network at least one bucking winding,
no further winding can be added.

13.   INTRODUCTION OF BUCKING WINDINGS

Experience in the construction of relay networks shows that
in some cases it is efficient to used so-called bucking windings.

In particular, the use of such windings makes it possible to
improve the time factors of a relay, and also replace certain con-
tacts in a relay circuit by their inverses.

We note that in general it is immaterial which of the windings
is the basic one and which is the bucking one. The basic winding
always is bucking with respect to the bucking winding, i.e.,

$$\bar{\bar{A}} \equiv A. \tag{8.39}$$

In the general case the ampere turns produced by ~~any~~ each
winding (both basic and bucking) can have different values and
their interaction may be sufficiently complicated.

We shall consider only networks in which the parameters are

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so chosen, that each winding can be in the following states:

a) "Excited," characterized by the coefficient $\left|\lambda'_{Ai}\right| \geqslant 1$, which is same for all windings.

$$\left|\lambda'_{A1}\right| = \left|\lambda'_{A2}\right| = \ldots = \left|\lambda'_{Ak}\right| \geqslant 1; \qquad (8.40a)$$

b) "Unexcited," characterized by a coefficient λ''_{Ai} such that

$$\left|\sum_i \lambda'_{Ai}\right| \leqslant \frac{AW_{oA}}{AW_{cA}}. \qquad (8.40b)$$

We shall confine ourselves here to such transformations, which do not change the values of $\lambda_A = \sum_i \lambda_{Ai}$ for all possible states of the network.

It is precisely for these conditions that we shall employ the distributive law of relay networks, formulated by D. I. Shnarevich /91/:

$$(x + y) A \equiv xA^1 + yA^2 + xy\overleftarrow{A^3},$$

$$xy + A \equiv (x + A^1)(y + A^2)(x + y + \overleftarrow{A^3}).$$

or

$$(8.41)$$

From this follow, for example, the following equivalences:

$$(x + y) A \equiv (x + \bar{x}y) A \equiv xA^1 + \bar{x}yA^2; \qquad (8.42)$$
$$A \equiv (x + \bar{x}) A \equiv xA^1 + \bar{x}A^2. \qquad (8.43)$$

In the particular case when $xy = 0$, i.e., $x \leqslant \bar{y}$ or $y \leqslant \bar{x}$, formulas (8.41) assume the form of the distributive law for networks without bucking windings.

Another transformation, which makes it possible to introduce into the network a bucking winding and retaining the values of λ_A,

by changing the structural admittance, is the equivalent transformation, recommended by A. N. Yurasov /64, 65/ and D. I. Shnarevich /91/:

$$x A \doteq A^i + \overleftarrow{x \, A^2};$$
$$x + A \doteq A^i (\overline{x} + \overleftarrow{A^2}).$$

$$(8.44)$$

We note that after introducing into the network bucking windings one can no longer use the transformations (8.38), which follow from the law (8.37) of repetition for networks without bucking windings, for in this case the relation is violated between the number of excited direct and bucking windings, and consequently λ_A changes.

If for any state η of the system relation (8.6b) is possible, indicating that relay A does not operate in this state either because of lack of current in the winding or because the magnetic fields produced by the individual windings are cancelled out, then each of the windings A^i of this relay can be considered only as a finite conductance $G^i_{= A^i}$ of the same order as the winding. Consequently, in particular, it follows that in the case when identical ampere turns are produced in the windings A^i and $\overleftarrow{A^i}$,

and

$$A^i + \overleftarrow{A^j} \equiv G_{= A^i, \overleftarrow{A^j}}$$
$$A^i \cdot \overleftarrow{A^j} \equiv G_{= A^i, \overleftarrow{A^j}}.$$

$$(8.45)$$

Using the equivalences (8.25), (8.45), and (8.43), one can

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obtain a derivation of the first of formulas (8.44)

$$xA \doteq xA + \overline{x}G_{=A} \equiv xA + \overline{x}(A + \widetilde{A}) \equiv$$
$$\equiv xA + \overline{x}A + \overline{x}\widetilde{A} \equiv A + \overline{x}A.$$

Analogously, the second formula can be derived.

14. GENERAL RULES FOR TRANSFORMATION OF RELAY NETWORKS

As already mentioned, all transformation of relay networks can be broken up into equivalent, in which the structural admittance changes, an absolute equivalent, in which only the action of the network is retained, but also the structural admittance. Certain transformations, such as formulas (8.38), for example, are absolutely equivalent when all the relay windings contained in them have the same order of conductance.

Inasmuch as in absolutely equivalent transformations the structural admittance of the circuit does not change, consequently its action on the other   networks   with which it may be connected does not change. One can therefore conclude that absolutely equivalent transformations can be used without any limitation both in the network as a whole, and in its individual parts.

The situation is different with transformations which result in a change in the structural admittance. These transformations can be₄ applied without limitation only to the network as a whole. When these transformations are applied to parts of networks, one

must verify how they influence the remaining parts of the network.
In the general case, to carry out transformations which are not absolutely equivalent on individual parts of the network, it is recommended that the necessary part be separated from the network, as indicated in the next chapter, the transformation carried out, after which this part is connected with the remaining network.

By way of an example of using transformations of the present chapter let us examine the derivation of formula (3.25) of the operation of a relay with bucking winding in a sequential network, as obtained by M. A. Gavrilov /16/ on the basis of a study of the operating conditions of sequential networks.

Starting with formula (3.20) for the operation of the intermediate relay A, the connection network of this relay can be written in the form

$$F = (r + a\bar{h})\, A.$$

(8.46)

where r and h are functions of operation and release of the network A.

Since the operating and release circuits cannot be simultaneously closed, $rh = 0$ or $\overline{rh} = r$. Taking this into account, formula (8.46) can be transformed in the following manner:

$$
\begin{aligned}
F &\equiv (r + a\bar{h})\, A \doteqdot (r + \bar{a}\bar{h})\, A + ah\,(A + \vec{A}) \equiv\\
&\equiv rA + a\bar{h}A + rah\bar{\bar{A}} + ah\,(A + \vec{A}) \equiv\\
&\equiv rA + a\bar{h}A + ar\vec{A} + ahA + ah\vec{A} \equiv\\
&\equiv rA + aA + ar\vec{A} + ah\vec{A} \equiv (r + a)\, A^1 + ah\vec{A}^2.
\end{aligned}
$$

Thus, a transition is realized from the network of Fig. 78a to the network of Fig. 78b. Analogously one can obtain other networks with bucking windings.
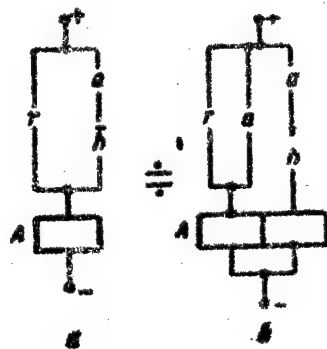
Fig. 78.

Chapter 9

CONSTRUCTION OF MULTIPLE-RELAY NETWORKS

1. General Remarks

In the synthesis of relay networks one obtains from the operating conditions of the network the operating conditions of the individual relays and actuating circuits in the form of analytical formulas or assemblies of numbers. In other words, in the first stage of the synthesis one obtains a group of networks of the form $f_A A$, $f_B B$, etc., for the operation of the intermediate relays A, B, ..., and $f_W W$, $f_V V$, ..., for the actuating relays V, W, ..., (Fig. 79).

The next stage is to construct the overall network, where it is necessary to attempt to obtain the simplest possible network, in particular, networks with the smallest number of contacts. The reduction in the number of contacts in the network is attained by unifying contact circuits acting on different relays. The possibility of unifying the circuits of separate relays is determined by the conditions imposed on the individual circuits.

It becomes possible most frequently to unify contact circuits which are connected to one of the terminals of the battery. One must attempt to have as many contact circuits as possible converge

at one point, for in this case the possibilities of unifying individual contacts increase.

In the present chapter we shall consider conditions, under which it is possible to unify individual networks, and methods of reducing the number of contacts in a network.

The unification of individual circuits in a common network is possible in two ways.

In the first method, which we shall call joining, the individual circuits are joined together in parallel or in series.

The second method, which we shall call combining, consists of reducing circuits of   separate relays into a single structure, after which relays from other networks are introduced in one of them.

The principal condition which determines the possibility of joining individual circuits into a common network is such a choice of parameters of all circuits, under which the states of one of the circuits does not influence the operation of the relays in other circuits. And since such an influence is determined above all by the admittances of the joined networks, we shall stop to discuss the character of the structural admittances of different relay circuits.

2. CLASSIFICATION OF RELAY NETWORKS BY THE CHARACTER OF THE ADMITTANCE

The structural admittance of the relay network is characterized

as the admittance of the individual windings of the relays and resistances, and by the methods of unifying the contact circuits with the windings and resistances.

In general one can subdivide all circuits into four groups.

1) Group A -- networks with finite admittance, characterized by the fact that in all cases their admittance have a finite value, i.e., they satisfy the inequality:

$$0 < Y < 1; \tag{9.1}$$

(Here zero corresponds to a closed circuit and one to an open circuit (admittance equals infinity).

2) Group B -- networks of normal type, characterized by the fact that the contact circuits are connected in series with elements of finite admittance. In such networks the admittance can never be unity, and satisfies the inequality

$$0 \leqslant Y < 1; \tag{9.2}$$

3) Group C -- networks of inverse type, where the contact circuits are connected in parallel with elements of finite conductivity, as a consequence such a circuit can never be opened and satisfies the inequality

$$0 < Y \leqslant 1; \tag{9.3}$$

4) Group D -- mixed networks; the contact circuits are connected both in series and in parallel to elements of finite conductivity and their admittance satisfies the inequality

$$0 \leqslant Y \leqslant 1. \tag{9.4}$$

3. Elementary Relay Circuits

A relay circuit which contains only one single-winding relay A will be called an elementary relay circuit. Fig. 80 shows elementary relay circuits of all four groups.

In the presence of several elementary relay circuits we shall compare them in order of conductance, meaning thereby the order of conductance of the relay windings contained in the particular circuit.

It is obvious that an elementary relay circuit of group A can never change its conductance. Elementary relay circuits of groups B and C may have two values of conductance: finite, equal to the conductance of the relay winding, and respectively zero or one. Circuits of group D can have both values zero and one, as well as finite values.

Elementary circuits of classes B and C can be transformed into each other by simple inversion, and can be transformed into networks of class D and vice versa by partial inversion. This does not violate the elementary nature and the operating conditions of the relay.

An elementary circuit of group D can be transformed into a network of group B or C by series or parallel connection of a finite conductance $G_{=A}$, as follows from transformations (8.27) -- (8.28):

$$\varphi + fA \doteq (\varphi + fA)\left(G_{=A} + \frac{\overline{\varphi}}{0}\right); \qquad (9.5)$$

$$\varphi + fA \doteq \varphi + fA + \frac{\overline{\varphi}}{1} G_{=A}. \qquad (9.6)$$
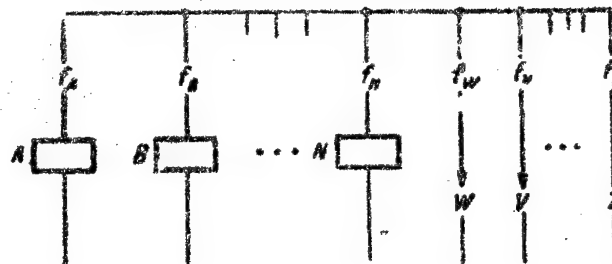
Fig. 79.
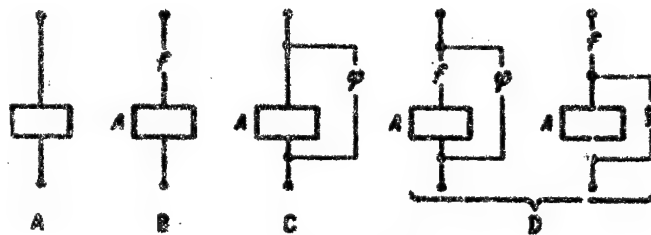
Fig. 80.
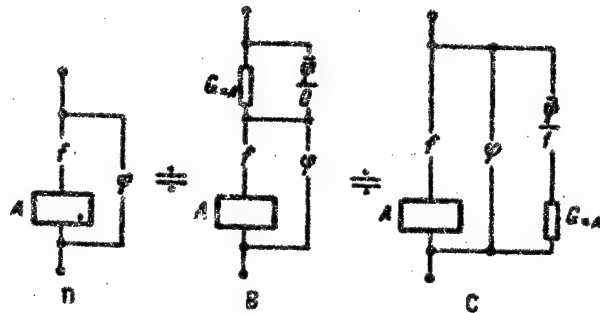
Fig. 81.

In turn, elementary circuits of group B and C can be transformed
into group A by analogous addition of an element of finite conduct-
ance or by introducing a bucking winding in accordance with formula
(8.44):

$$xA \doteqdot xA + \frac{\overline{x}}{1} G_{=A};$$
$$x + A \doteqdot (x + A)\left(\frac{\overline{x}}{0} + G_{=A}\right);$$

$$(9.7)$$

$$xA \doteqdot A + \overline{\overline{xA}};$$
$$x + A \doteqdot A(\overline{x} + \overleftarrow{A}).$$

$$(9.8)$$

An elementary relay circuit of group A cannot be transformed
into any other group.

4. CONDITIONS OF JOINING RELAY CIRCUITS INTO A COMMON
    NETWORK

When joining several relay circuits into a common one it is
necessary to   satisfy the condition whereby the states of
some circuits do not affect the operation of the relay in other
circuits.

Starting with the foregoing, we can see that when several
relay circuits are connected in series the conductance of each of
these should be less in order of conductance than all the remaining
ones. To the contrary, in parallel connection each conductance
should not be of greater order.

It is obvious that the joining of several circuits, satisfying
this condition, is possible only within the groups A, B, and C for
identical orders of relay conductances in each circuit, wherein:

1) The circuits of group A can be joined both in parallel and

in series, as can be done with elements of finite conductance of the same order, and also connected to circuits of group B in parallel or to   circuits of group C in series.

2) Circuits of group B can be joined only in parallel with each other and with circuits of group A.

3) Circuits of group C can be joined only in series with each other or with circuits of group A.

4) Circuits of group D cannot be joined either in series or in parallel with any other circuits.

We see that the most "flexible" from the point of view of joining with other circuits are circuits of group A. Circuits of groups B and C are equivalent, while circuits of group D are the most unsuitable.

Elementary relay circuits of different orders of conductance cannot be interconnected into a common network.

The most practical significance is a parallel connection of normal elementary relay circuits (group B). Here one should tend towards unifying contacts which enter in circuits of different relays. For this purpose   formulas of individual circuits are reduced to such a form so as to be able to take outside the brackets the largest number of elements. As a result of such transformations one obtains again a circuit of normal type. It is possible to construct circuits by graphical methods, if the conditions of operation of individual relays are specified by

assemblies of numbers.

One join analogously inverse networks (group C).

5. Breakdown of Elementary Relay Circuits

In some cases it is advantageous to breakdown the elementary relay circuit, before joining it to other circuits, into several smaller relay circuits, by introducing additional windings. Such a breakdown can be obtained by introducing a coordinated winding in accordance with formulas (8.38) and (8.42) or bucking windings in accordance with formulas (8.41) and (8.44). After breakdown, each elementary circuit can be considered independently, bearing in mind only that when a bucking winding is introduced in any particular relay, further introduction of coordinated windings in accordance with formula (8.38) is impossible.

The breakdown is best carried out in such a way that in individual circuits the contact part of the network will be if possible the same structure, or   else that they have the maximum number of identical contacts, which could be brought out into a unified circuit.

Let us consider, for example, the transformations of a network of a binary pulse divider, the circuits for whose relays A and B are specified by the formulas

$$f_A = u\overline{b} + \overline{u}a:$$
$$f_B = \not{a}b + \overline{u}a.$$

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Introducing combined windings, the circuit can be represented in the form of four elementary circuits of group B: $i\overline{b}A^1$, i_bA^2, ibB^1, and i_aB^2. After such a breakdown, it is advantageous to unify windings A^1 and B^2 into common networks, as well as A^2 and B^2, after which the structural formula of the network becomes

$$F = u(\overline{b}A^1 + bB^1) + \overline{i}aA^2B^2,$$

where the winding should be of the same order of conductance.

This circuit can be transformed further by introducing bucking windings, to such a form as to have, for example, one make contact on relay I. Using transformation (8.44), we replace the contact $\overline{i}$ by i, i.e., we write $\overline{i}aA^2B^2 \doteq i_aA^3B^3 + aA^2B^2$, and consequently we obtain the circuit

$$F = u(\overline{b}A^1 + bB^1 + a\overleftarrow{A}^3\overrightarrow{B}^3) + aA^2B^2,$$

represented in Fig. 26a. In this circuit all the windings are of the same order of conductance, and the windings $A^2 - A^3$ and $B^2 - B^3$ should produce the same number of ampere turns.

6. COMBINING RELAY CIRCUITS

The joining of elementary relay circuits, as indicated above, leads either to a normal network or to an inverse one. The probabilities are much greater if one goes to mixed networks with parametric action on individual relays.

In joining relay circuits, it is possible to bring out any

contact (or contact circuit) into a common circuit provided this contact (circuit) is in series with the circuit of the relays that are joined together.. It is impossible to use one and the same contact, for example, to act on two relays if the operating formula of one relay contains a make contact and the formula of the other contains a break contact, with a simple joining of the relay circuits. In the mixed circuit, however, if this contact is connected in parallel to one relay and in series to another, its actions on these relays will be opposite.

To construct mixed relay networks we use the equivalent transformations indicated in Chapter 8, together with the possibility of introducing into relay circuits elements of finite conductivity. The circuit of each relay (or each of the relay windings) will be transformed in such a way as to reduce them to a unified structure, i.e., to circuits with identically located like contacts and elements of finite conductivity. These circuits of unified structure can then be combined, i.e., individual elements of finite conductivity can be replaced by corresponding relay windings from different circuits with observation of the necessary orders of conductivity of these windings.

By way of an example let us consider the transformation of a network, the synthesis of which is carried out in reference 73, pp 534 --- 535, in which the functions of the operation of the relays X_1, X_2, and X_3 are given by the formulas

$$\left.\begin{array}{l} f_{x_1} = a + x_1\bar{x_2}\bar{x_3}; \\ f_{x_2} = \bar{a}x_1 + x_2(x_1 + \bar{x_3}); \\ f_{x_3} = ax_2 + x_1x_2. \end{array}\right\}$$

(9.9)

We consider first networks which contain the contacts of relay A. For this purpose we provide each relay with two windings and transform the expression to a unified structure:

$$F_1^1 \doteqdot aX_1^1 \doteqdot (a + \omega G_{<x_i^1})X_1^1 \doteqdot (a + \omega G_{<x_i^1})(X_1^1 + \omega G_{<x_i^1});$$
$$F_2^1 = \bar{a}x_1X_2^1 \doteqdot a + x_1X_2^1 \doteqdot (a + x_1X_2^1)G_{>x_i^1};$$
$$F_3^1 = ax_2X_3^1 \doteqdot (a + \omega G_{<x_i^1})x_2X_3^1 \doteqdot (a + \omega G_{<x_i^1})(x_2X_3^1 + G_{<x_3^1}).$$

(ω denotes any contact circuit).

Comparing the three resultant expressions we can note that the first factor of each has a structure $a + x_1 X_2^1{}_{<x_2^3 x_3^2}$, while the second is an element of finite conductivity of form $(x_1^1 \leqslant x_3^1 + x_2 x_3^1 \leqslant x_2^1)_{>x_2^1}$. From this we conclude first that X_1^1 and X_3^1 should be of the same order. From a comparison with the first factor it follows that the order of the conductance of X_2^1 should be less than the order of the conductances of X_1^1 and X_3^1.

Consequently, we obtain finally a structural formula
$$F^1 = (a + x_1X_2^1{}_{<x_1^1 \cdot x_3^1})(X_1^1 + x_2X_3^1).$$

We note that this network has the character of a normal relay network. In the calculation of the relay windings it is necessary to take into account that the winding of relay X_2^1 should have such a resistance, that when it is connected in series

with relay X_1 or X_3 it does not hold; relay X_2 should operate when connected in series with the winding of relay $X_1{}^1$.

We now transform the second parts of the formula, containing the contacts of the relays $X_1 X_2$ and X_3:

$$F_1^2 = x_1 \bar{x}_2 \bar{x}_3 X_1^2 \doteq \bar{x}_2 \bar{x}_3 (\bar{x}_1 + X_1^2) \doteq (\bar{x}_3 + \omega G_< x_1^2)(\bar{x}_2 + G_< x_1^2)(\bar{x}_1 + X_1^2);$$
$$F_2^2 = x_2 (x_1 + \bar{x}_3) X_2^2 \doteq (x_1 + \bar{x}_3)(X_2^2 + \bar{x}_2) \doteq (\bar{x}_3 + x_1 G_> x_1)(\bar{x}_2 + X_2^2)(G_> x_2^2 + \omega);$$
$$F_3^2 = x_1 x_3 X_3^2 \doteq \bar{x}_3 + x_1 X_3^2 = (\bar{x}_3 + x_1 X_3^2)(G_> x_3^2 + \omega).$$

From a comparison of the individual parts of the resultant expressions we can write for the first part

$$\bar{x}_3 + x_1 X_3^2 {}_< x_1^2 {}_\gg x_2^2$$

and for the last parts

$$[(\bar{x}_2 + X_2^2 {}_< x_1^2)(\bar{x}_1 + X_1^2 {}_\gg x_2^2)] {}_\gg x_3^2 .$$

Comparing the orders of the conductances, we see that $X_2{}^2$ and $X_3{}^2$ are of the same order, while $X_1{}^2$ is of a larger order, i.e., finally

$$F^2 = (\bar{x}_3 + x_1 X_3^2)(\bar{x}_2 + X_2^2)(\bar{x}_1 + X_1' {}_> x_2^2 x_3^2).$$

In order to be able to join the networks F_K^1 and F_K^2 in parallel, the latter should be reduced to one of group B by connecting in series a conductance R, the order of which should be such that:

The operation of the relay of network F^2 must not be disturbed, for which purpose it should be greater than or equal to the relay with the highest order of conductance in this network, i.e., $R \gg X_j^2$.

The operation of the network F^1 must not be disturbed, for

which purpose it should be equal to or less than the conductance of the relay of network F^1 having the lowest order, i.e., $R \leqslant \chi_2^!$.

The final network will be as shown in Fig. 82a.

To satisfy the conditions that follows from the foregoing relations for the orders of the conductance, the windings of the relay must be so designed, that relay X_1 does not hold when the windings of X_2^2 or X_3^2 are connected in series, and the relays X_2 and X_3 operate when all three windings together with the limiting resistance R are connected in series.

A shortcoming of this network is the fact that in quiet state the circuit is continuously closed. In order to eliminate this we take the inverse network of F^2:

$$\overline{F}^2 \doteq x_3 (\overline{x}_1 + X_3^2) + x_2 X_2^2 + $$
$$+ x_1 X_1^2 < x_1^! \cdot x_1^!$$

Here, too, we must use a limiting resistance R, and the winding of the relay should be so designed that the relay X_1 does not hold when connected in parallel with winding X_1^2, winding X_2^2, or winding X_3^2, while the relays X_2 and X_3 should operate when all three windings are connected in parallel.

The new network is shown in Fig. 82b. The resultant networks have two or three contacts (four or five springs) less than networks assembled by usual synthesis methods /73, Fig. 15/.

By way of another example let us consider the synthesis of
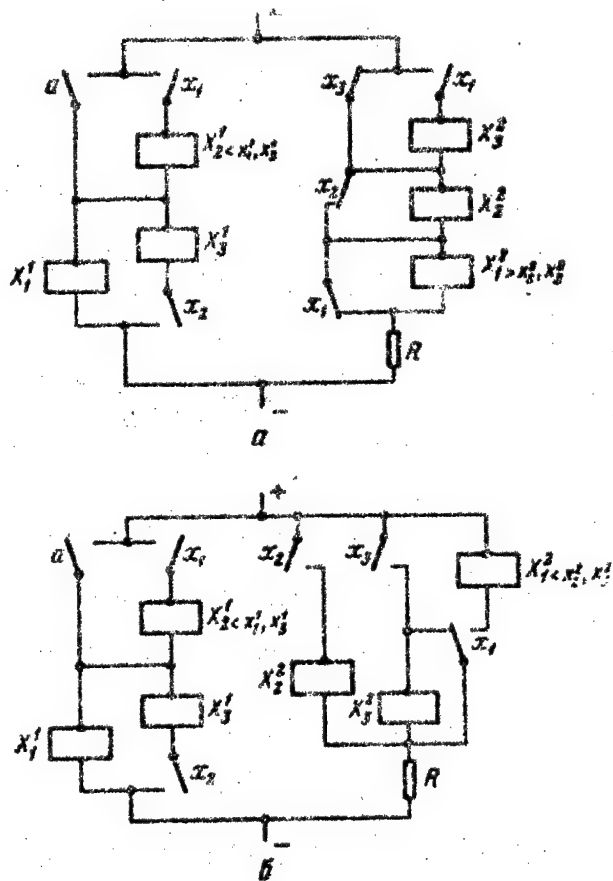
Fig. 82.
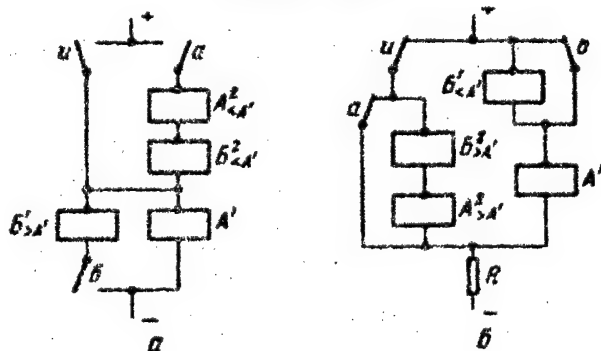
Fig. 83.

a well known two-relay network of a binary divider for ~~xx~~ a number
of pulses, the structural formula of which has the form

$$F \doteq u\bar{6}\bar{A}^1 + u6\bar{5}^1 + \\ + \bar{u}aA^2\bar{5}^2.$$

(9.10)

In analogy with the preceding, we transform the individual
terms of this expression

$$F_1 = u\bar{6}A^1 \doteq u\,(6 + A^1) \\ \doteq u\,(6G_{>A^1} + A^1) \doteq \\ \doteq (u + {}_\omega G_{<A^1})(6G_{>A^1} + A^1);$$

$$F_2 = u6\bar{5}^1 \doteq u\,(6\bar{5}^1 + G_{<\bar{5}^1}) \doteq (u + \omega G_{<\bar{5}^1})(6\bar{5}^1 + G_{<\bar{5}^1});$$
$$F_3 = \bar{u}aA^2\bar{5}^2 \doteq u + aA^2\bar{5}^2 \doteq (u + aA^2\bar{5}^2)\,G_{\geqslant A^1,\bar{5}^1}.$$

Comparing the individual expressions we can conclude that
the general structural formula can be written in the form

$$F = [u + a\,(A^2\bar{5}^2)_{<A^1}\,](6\bar{5}^1_{>A^1} + A^1).$$

The corresponding network is shown in Fig. 83a.   Inverting
this form, we obtain a new network (Fig. 83b):

$$F = \bar{u}\,[\bar{a} + (A^2\bar{5}^2)_{>A^1}\,] + (\bar{6} + \bar{5}^1_{<A^1})\,A^1.$$

In the first network the first windings of relays A and B
and the limiting resistance should be so designed that when
winding B^1 is connected in parallel to winding A^1 relay A releases,
and relay B operates. The second winding A^2 and B^2 should be so
designed that both relays operate when these windings are
connected in series with winding A^1 and the limiting resistance.
In the second network (Fig. 83b), the winding B^1 should be so de-
signed that when connected in series with the winding A^1 relay

A releases and B operates.

We note that these networks have one contact only on each relay, and consequently, it is impossible to obtain a network with fewer contacts.

The solutions given here are not unique, since one can obtain a large number of analogous networks of different structures.

Chapter 10

SEQUENTIAL RELAY NETWORKS WITH CAPACITORS

1. The Capacitor -- An Active Element of Relay Networks

In relay networks capacitors are widely used principally to produce delayed operation and release of relays, sometimes to accelerate the operation, and for suppression of sparks. The ability of a capacitor to accumulate and store charge and to produce short-duration current pulses during the time of charging or discharging are used considerably less frequently.

These properties make it possible to use the capacitor as a memory element in a relay sequential network, which may operate on the relay /50/. In the general case the capacitor can have different charges, but from the point of view of operation of a relay network, we are interested in only two states of the capacitor -- discharge and charge with a voltage capable of acting on the relay.

Inasmuch as the processes of charging and discharging of capacitors in different circuits have been studied quite well, we shall stop here only to discuss problems of synthesis of networks with capacitors acting on relays.

Fig. 84 shows the simplest way of acting on intermediate relay
X through contact i of the receiving element. Relay X operates only after
the receiving element operates and then releases. If the relay has
a device for holding (polarised relay or a relay with self-
holding, and also a holding winding /dotted in Fig. 84/, it will
remain in operating condition.

The relay can be switched off either by breaking the holding
circuit, or by transmitting a current pulse from a capacitor in
a bucking winding, as shown in Fig. 85, where the circuits r and h
correspond to states of the network in which the relay X should
operate (r) or release (h).   One must connect in a circuit
of the bucking winding a make contact of this relay, so that after
this relay releases the relay does not operate again because of
current in the bucking winding.

Thus, to operate a relay without holding it is necessary to
have three windings -- working, holding, and bucking. We note that

the relay can be held with the working winding, if the
supply is connected to it through a resistance R by means of a
transfer contact on the same relay. In this case the parameters
of the circuit should be so chosen that the capacitor does not
have time to become charged during the instant of transfer of the
contact, when the extreme springs are interconnected. It is
possible to eliminate this effect by introducing a rectifier
element B in the charging circuit of the capacitor (see Fig. 85).

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Fig. 84.

Fig. 85.
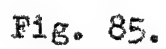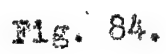
Fig. 86.

Since the duration of the pulse is limited, one must choose such a duration, that one relay has a chance to operate or release, and the other, connected in parallel to the contact of the operating (releasing) relay does not operate. Under this condition one can use one capacitor to control several relays.

From an examination of the networks we see that corresponding to charged and uncharged states of the capacitor (in analogy to the operating and non-operating states of the relay) are individual periods of operation of the network. As regards its action on other elements of the network, the capacitor differs from the relay. After the relay operates, it switches circuits (one or several) and after causing a certain action in other elements of the circuit, it can in itself not change its own state. On the other hand the action of the capacitor can take place only during instants of charging or discharging, i.e., the action   is accompanied by a change in the state of the capacitor.

Thus, the capacitor can be used only in these cases when it is enough to have a short-duration action, as for example to actuate an intermediate relay. The possibility of having a capacitor act on actuating circuits is very limited and requires investigation in each particular case.

As will be shown below, the use of capacitors in sequential networks makes it possible to reduce the number of relays, which in some cases leads to an increase in the number of

contacts and windings in the remaining relays. Sometimes the use of capacitors makes it possible to solve problems of interaction between the intermediate relays, by realizing charges and discharges in a corresponding sequence, and also by transferring the charge from one capacitor to another.

In some cases an effective solution can be obtained by combining the action of both the capacitor and purely contact networks on a given relay. Thus, for example, one can let the relay operate by a capacitor and release by breaking the holding circuit.

The synthesis of relay networks with capacitors can be carried out using methods   used for the synthesis of relay circuits, and the purely contact-making multipole can be separated in the network with capacitors.

2. Synthesis of Sequential Networks with Capacitor Acting on All the Relays.

For the synthesis of networks with capacitors, we can use methods used in the theory of contact networks for the synthesis of purely relay networks.

Let us first consider the synthesis of a network with a single capacitor acting on   n relays of the network. Such a network can be constructed in such a way that the capacitor is first charged and then discharged through the winding of the relay through a corresponding contact circuit. In addition, it is

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necessary to provide a circuit for holding the relay. Applying the

synthesis procedure of Chapter 3 to networks with capacitors,

we shall use connection tables to determine the necessary number of

intermediate elements and the instants during which their states

must change.

If it is considered that a capacitor can have two states —

uncharged and charged — then it can be used as one of the circuit

elements. In this case the connection table must be constructed in

such a way, that during the period of network operation, which

differ only in the fact that the capacitor is charged and dis-

charged, the state of the actuating relays be

the same.

We introduce into the connection table a separate row, in

which an arrow ($\longrightarrow$) will denote the instant of capacitor charging

and a cross ($\times$) the steps during which the capacitor retains

its charged state. The charge of the capacitor will take place

during the steps when the relays, on which the capacitor acts,

operate or release. Thus, the connection tables will note in

addition the steps of capacitor charging. The steps will

be numbered as in tables without capacitors, designating

with a cross the numbers of those steps, in which the capacitor

is charged. The difference between all numbers (including the

cross) indicates the possibility of realization of such a network.

In accordance with the foregoing, the table of connections

for a counting network for four pulses, in which the actuating circuits are closed only after the release of the receiving pulse relay I, will have the form shown in Fig. 86.

After the table of connections is made up, we proceed to synthesize the network.

Since in the process of operation of the network the circuit f of the capacitor should be closed during the charging steps, and the corresponding relay operation and release steps should cause the closing of the capacitor charging circuit to the working or bucking windings of the relays, a sequential n-relay network with capacitor can be represented in the form of a contact multipole (Fig. 87) with one input and kxxx 2n + 1 outputs. Connected to the input of the multipole is a capacitor, and to one output is connected one terminal of the battery through a limiting resistance R, while the working and bucking windings of the relays are connected to the remaining outputs (the holding circuits are not shown in the diagram, but if the relays are unpolarized they should be provided with such circuits). The output to which the battery terminal is connected should be isolating as regards to all remaining outputs.

From the connection table we find the following.

The formula for the capacitor charging circuit f_ξ as the sum of the constituents corresponding to the charging steps (designated by an arrow in the row ξ).

The formula f'_X of the working winding of relay X as the sum

Fig. 87.

Fig. 88.

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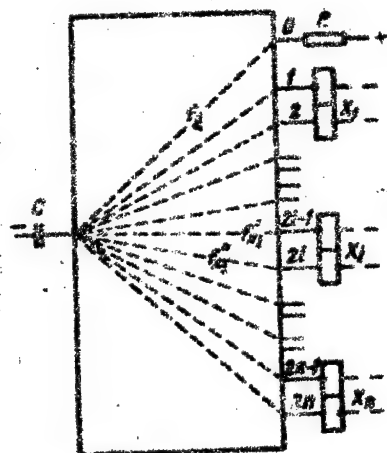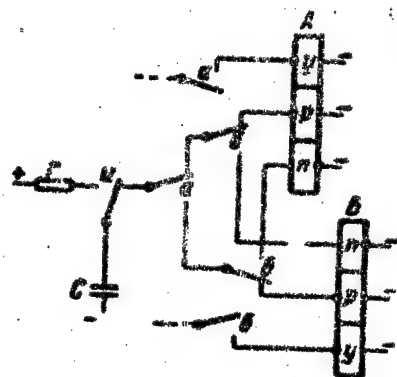
of the constituents corresponding to the steps of operation of this relay (marked with an arrow in the table).

The formula f''_x of the bucking winding of the same relay X as the sum of the constituents corresponding to the release steps (marked with the symbol ⊣).[9] In this case the conditional terms in formulas f_ξ , f'_X and f''_X can be taken to be the constituents corresponding to the unused states, and in formulas f'_X and f''_X also the constituents of those states of the network, in which the capacitor acting on the given circuit is not charged.

If we denote by E the terminal of the battery   with which the capacitor is charged (with resistor R), then the structural formula of the multipole network (without the holding circuits) can be written in general form

$$F = f_i E + \sum_{i=1}^{n} (f'_{x_i} X_i + f'_{x_i} \bar{X}_i). \qquad (10.1)$$

We note furthermore that in the synthesis of a relay network with capacitors it is essential that the capacitor return to its initial state after the completion of the operation. This may require additional devices for charging the capacitor at the end of operation of the network, if it remained in charge state.

By way of example let us consider a construction of a simplest counting network with two counting relays (A and B) and a capacitor, operating in accordance with the connection table indicated in Fig. 86.

Starting with this table, we obtain

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$$f_t = \bar{u}\bar{a}\bar{b} + u a \bar{b} + u a b + \bar{u}\bar{a}b = u;$$
$$f'_A = \bar{u}\bar{a}\bar{b};$$
$$f^*_A = \bar{u}a b;$$
$$f'_B = \bar{u}\bar{a}\bar{b};$$
$$f^*_B = \bar{u}\bar{a}b.$$

Consequently

$$F = uE + \bar{u}\bar{a}\bar{b}A + \bar{u}a b \bar{A} + \bar{u}\bar{a}\bar{b}B + \bar{u}\bar{a}b\bar{B} =$$
$$= uE + \bar{u}\,[\bar{a}\,(\bar{b}A + b\bar{B}) + a\,(\bar{b}\bar{B} + b\bar{A})].$$

The corresponding network with the holding circuits added is shown in Fig. 88.

3. Network with Decoupling Capacitors.

In the networks shown above, one capacitor acted on several relays, and this called for a suitable calculation of the duration of the pulse and introduction of separate contacts for each relay circuit.

The network can be made to operate more stably when different capacitors, charged during different times act on the different relays or windings. Thus, Fig. 89 shows a connection table for a network with two relays, operated by two separate capacitors.

In this case the capacitor charging circuits will be:

$$f_{t_1} = (1,7) = u\bar{a}\bar{b} + u a b = u\,(\bar{a}\bar{b} + a b);$$
$$f_{t_2} = (3,5) = u a \bar{b} + u\bar{a}b = u\,(a\bar{b} + \bar{a}b).$$

Fig. 89.

Fig. 90.

In the synthesis of circuits for relay A we can take as the conditional terms the constituents corresponding to all steps, in which capacitor C_1, acting on this relay, is not charged, i.e., with numbers 2, 3, 4, and 5.

Taking this account we obtain for relay A

$$f'_A = \{0, (2, 3, 4, 5)\} = \bar{u}\,\frac{\bar{a}}{6} + \frac{u\,(\overline{a6} + \overline{a6})}{0};$$

$$f''_A = \{6, (2, 3, 4, 5)\} = \bar{u}\,\frac{a}{6} + \frac{u\,(\overline{a6} + \overline{a6})}{0}.$$

Analogously we obtain for the circuits of relay B

$$f'_B = \{2, (0, 1, 6, 7)\} = \bar{u}\,\frac{a}{6} + \frac{u\,(\overline{a6} + a6)}{0};$$

$$f''_B = \{4, (0, 1, 6, 7)\} = \bar{u}\,\frac{\bar{a}}{6} + \frac{u\,(\overline{a6} + a6)}{0}.$$

For the circuits of relays A and B we can take any one of the solutions, but it is most advantageous not to connect in the circuit of the working winding a contact of the given relay (this will result in total discharge of the capacitor), but to place in the circuit of the bucking winding a break contact of this relay. Accordingly, we obtain the network of Fig. 90.

It is also possible to employ a separate capacitor for each winding of relays A and B, as shown in the connection tables on Fig. 91.

Since the capacitor C_1 is not charged in all the states of the network, with the exception of zero and one, consequently all the remaining constituents can be taken as conditional terms for the circuit f etc. The corresponding network is shown in Fig. 92a. By unifying the contact (for which it is convenient

to introduce rectifier elements /16/, we obtain the circuit of Fig. 92b.

In some cases networks with capacitors can be simplified by separating the intermediate elements into individual groups in such a way, that the operation of the first group depends on the receiving elements, the operation of the second group depends on the first, etc. in analogy as it is advantageous to do in relay networks.

4. ADDITIONAL CAPABILITIES OF NETWORKS WITH CAPACITORS

The use of capacitors in   sequential networks makes it possible to solve also several other problems without introducing additional relays. Thus, if a charged capacitor is discharged before it is connected to the relay winding, this relay will not operate. Consequently, in the network of Fig. 93 the relay X will operate when the circuit f_X is close, provided the circuit f_ξ is closed earlier and the circuit $f_{\bar{\xi}}$ is not closed after this. A limiting resistance R should be connected in this circuit.

In the general case,   a relay network with such a circuit can be represented in the form of a multipole (Fig. 94), analogous to the multipole of Fig. 87 with one input and $\pm 2n + 2$ outputs.

In addition to producing a pulse for relay operation through discharge, one can use the process of charging the capacitor (Fig. 95). For normal operation it is necessary in this case to insure prior discharge of the capacitor through a resistance R.

The contact multipole is represented in this case in Fig. 96. We see that it differs from the multipole of Fig. 94 only in that the battery terminals are connected, and the synthesis of the network is analogous.

In some cases one can use both processes. Thus, for example, Fig. 97 shows a network in which the relay X operates everytime that the state of the contact u changes (once because of charging of the capacitor, and the second time because of the discharge).

Finally, circuits with capacitors have additional capabilities realized through the transfer of the charge from one capacitor to another. For example, in the network of Fig. 98a the relay X is fed when the circuits f_1, f_2, and f_3 are connected in sequence, and simultaneous and repeating closures of the circuits are permissible. In the circuit of Fig. 98b a disturbance to the sequence in closing of the circuit makes it impossible for relay X to operate.

Thus, for example, the circuit of Fig. 99 permits action on the relay X only if the relays A, B, and C operate and release in sequence. Any other sequence of the operation of these relays will not affect the relay X. If this problem is to be solved by means of relays alone, three intermediate relays must be used.
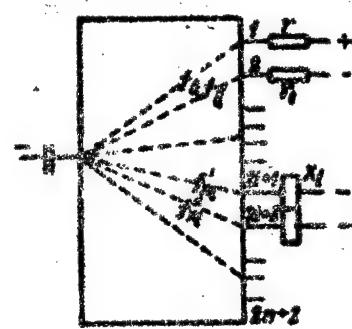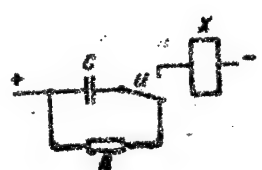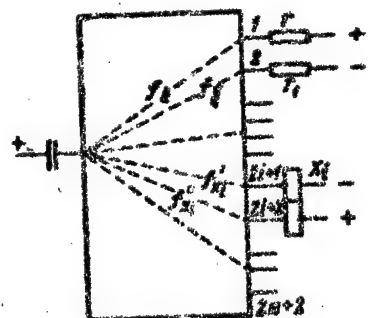
Fig. 91.

a

б

Fig. 92.
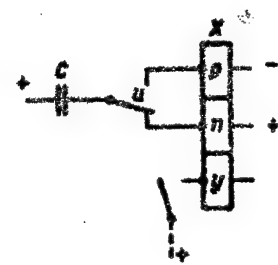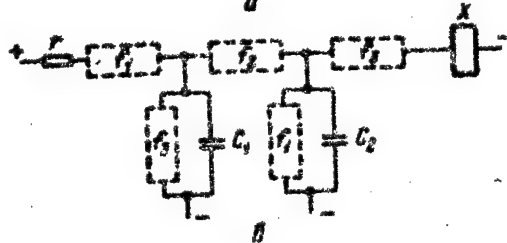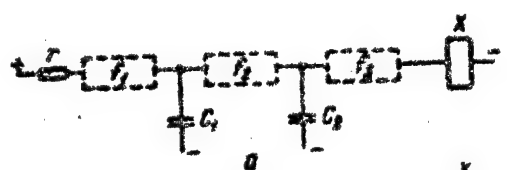
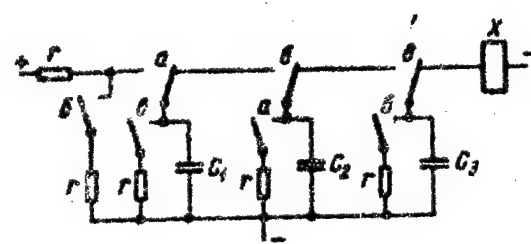Fig. 93.

Fig. 94.

Fig. 95.

Fig. 96.

Fig. 97.

Fig. 98.

Fig. 99.

Chapter 11

MECHANIZATION OF THE PROCESS OF STRUCTURAL
SYNTHESIS OF RELAY NETWORKS

1. GENERAL INFORMATION ON THE AUTOMATIZATION
OF THE SYNTHESIS PROCESS

The process of synthesis of relay networks is
quite time-consuming. The process is particularly
complicated because the same conditions correspond to
a large number (theoretically infinite) of different
systems, differing in the number of relays, their
sequence of operation, methods of constructing individ-
ual circuits, and other factors. The construction of
versions of the network and the choice of the optimum
call for a large amount of cumbersome work. The desire
to make the designer's work easy in the choice of
network versions has led to a search for the possibility
of automatization of the process of structural synthesis.
Attempts at mechanizing   algebraic methods of

network synthesis were unsuccessful, because of the
lack of a sufficiently good single-valued algorithm
for obtaining the structure of the network.

combination

A method developed at the Institute of Mathematical
Machines of the Czechoslovak Academy of Sciences
has resulted in a semi-automatic machine /78/ for the
construction of contact (1,k)-poles specified in terms
of structural admittances. This machine analyzes the
specification inserted in it and indicates which contact
must be placed in the network between the chosen term-
inals. The network is assembled manually, and the
machine determines whether the contact has been added
to the network correctly. As a result, a network
version is obtained which experience has shown to be
close to optimum as regards the number of contacts.

The graphic method (Chapter 7)   developed
in the Laboratory on Scientific Problems of   Wire
Communication, Academy of Sciences U.S.S.R., has made
it possible to develop a machine (suggested by V. N.
Roginskiy, V. G. Lazarev, and A. A. Akhangel'skaya /132/),

which automatizes the construction of several versions
of contact (1,k)-poles from assembles of obligatory
and conditional numbers, and also the choice of the
most suitable version for a specified minimum contact
or the distribution of contacts over the relays. The
machine simulates the basic operations of the graphic
methods, and the synthesized network is obtained on a
dummy panel.

Next, on the basis of the work by   V. G.
Lazarev /92/ on the determination of the minimum number
of intermediate relays necessary to realize a specified
sequence of actions (see Chapter 3, Section 4), it
became possible to automatize also the process of
constructing the connection table /112/.

In the present chapter we give brief information
on the principles of constructing the principal units
of this machine.

2. SIMULATION OF THE OPERATION OF THE GRAPHIC METHOD

In the graphic method (Chapter 7) for the

construction of contact networks, the following operations are performed:

1) Conditions are specified for each circuit in the form of assemblies of obligatory and conditional numbers.

2) Individual assemblies are compared with each other and those coinciding are detected.

3) The coinciding assemblies are unified and the common point is assigned the unified assembly.

4) The assemblies are separated by connecting to the point a relay transfer contact.

5) The necessary junctions and contacts are drawn and erased.

It is possible to indicate here the operating sequence in each stage. These properties of the graphic method have made it possible to develop a machine for the synthesis of contact (1,k)-poles. To produce a machine it is necessary to provide an electric model for each operation. Let us analyze the possibilities of simulation.

1. Specification of Assemblies

The conditions for each circuit are specified in the form of assemblies of obligatory and conditional numbers. In a network of n relays, the total number of numbers will be 2^n.

The simplest method of specifying assemblies is to assign to each number a separate wire and to apply to this wire different voltages depending on whether this number is obligatory or conditional. In this case to each output of a (1,k)-pole network one must assign a bundle of 2^n wires numbered from 0 to $2^n - 1$ and 2^n 3-position switches (or other types of transfer switches). Depending on the position of the switch, the corresponding wire receives either the total voltage (if the given number is obligatory) or half the voltage (if this number is conditional), or no voltage (for forbidden numbers), as shown, for example, in Fig. 100.

Other methods of specifying the assemblies can also be developed, for example in the form of pulses separated in time, etc.
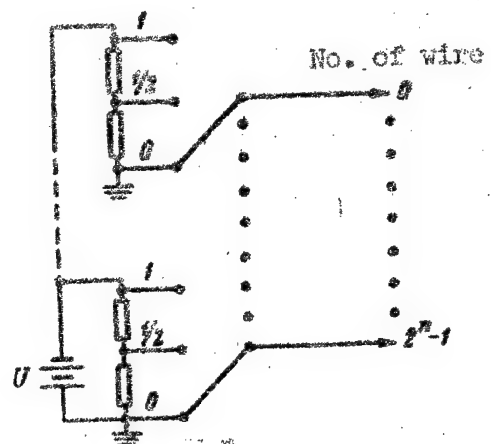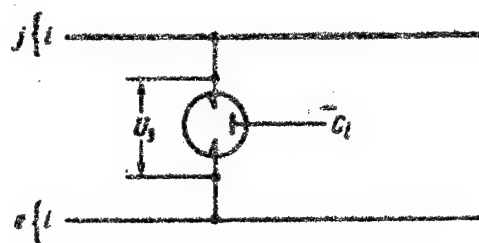
Fig. 100.

Fig. 101.

2. Comparison of Assemblies

and Establishment of Coincidences

The determination of coincidences of individual assemblies should be carried out with a special device connected ultimately to each pair of bundles.

To determine the coinciding assemblies, we introduce, on the basis of their definition (Chapter 6, Section 3), a coincidence function $C(f_1, f_2)$, which assumes a value 1 when the assemblies f_1 and f_2 coincide, and 0 otherwise, that is,

$$F \quad C(f_1, f_2) = \begin{matrix} 1, \text{ if } f_1 \text{ and } f_2 \text{ coincide} \\ 0, \text{ if } f_1 \text{ and } f_2 \text{ do not coincide} \end{matrix} \qquad (11.1)$$

To establish coincidence, it is necessary to compare all the numbers of the assemblies and two numbers will coincide if and only if a "unique coincidence" is established for each number, i.e., it is established that not one of the obligatory numbers of one assembly is contained among the forbidden numbers of the second assembly. The functions of the unique coincidence C_1

for each number i (where i = 0, 1, 2, ..., $2^n - 1$) should, in accordance with the definition of the coinciding assemblies, satisfy Table 21.

<table>
<tr><td rowspan="2"></td><td colspan="4" align="center">Table 21</td></tr>
</table>

β_i	α_i		
	0	1/2	1
0	1	1	0
1/2	1	1	1
1	0	1	1

β_i	α_i		
	0	1/2	1
0	0	0	1
1/2	0	0	0
1	1	0	0

Table 22

Then

$$C(f_\alpha, f_\beta) = c_0 \cdot c_1 \ldots c_{2^n-1} = \prod_{i=0}^{2^n-1} c_i. \qquad (11.2)$$

It is advisable to simulate not the coincident function itself $C(f_\alpha, f_\beta)$, but its inversion $\overline{C}(f_\alpha, f_\beta)$:

$$\overline{C}(f_\alpha, f_\beta) = \overline{c}_0 + \overline{c}_1 + \ldots + \overline{c}_{2^n-1} = \sum_{i=0}^{2^n-1} \overline{c}_i. \qquad (11.3)$$

where $\overline{c}_i$ is determined from Table 22.

For the numbers corresponding to voltages 0, ½ and 1, specified above, the function c_i can be readily realized by means of a thyratron, with a firing voltage located halfway between half and full voltage (Fig. 101):

$$\tfrac{1}{2} < U_s < 1. \tag{11.4}$$

The non-coincidence function $\overline{C}(f_k, f_\ell)$ of two assemblies is realized here by means of a parallel connection of the outputs of thyratrons connected to like wires of two bundles, as shown in Fig 102.

The sequence of comparison should be programmed beforehand, and primarily a comparison and unification should take place of bundles pertaining to each transfer contact, followed by the remaining bundles.

3.   Simulation of Unification

Two bundles of wires, between   which coincidence is established ($C(f_k, f_\ell) = 1$) should be unified, that

is, one bundle of wire should be obtained instead of two. The voltage on wire i of this bundle should satisfy Table 23 (in accordance with Chapter 6, Section 3).

Table 23

β_i	a_i		
	0	1/2	1
0	0	0	—
1/2	0	1/2	1
1	—	1	1

The simplest method of realizing the unification is direct connection of like wires of the unified bundles.

If a voltage 0 or 1 are applied to the wires directly, and a voltage ½ is applied through a limiting resistance, then after the unification the resultant voltages will satisfy Table 23. Since the result should leave only one bundle, the circuits in the wires in the second of the unified bundles should be disconnected.

Fig. 103 shows a joining scheme with the aid of a unification relay $C_{k\ell}$, which operates after a coincidence between bundles k and ℓ has been established.
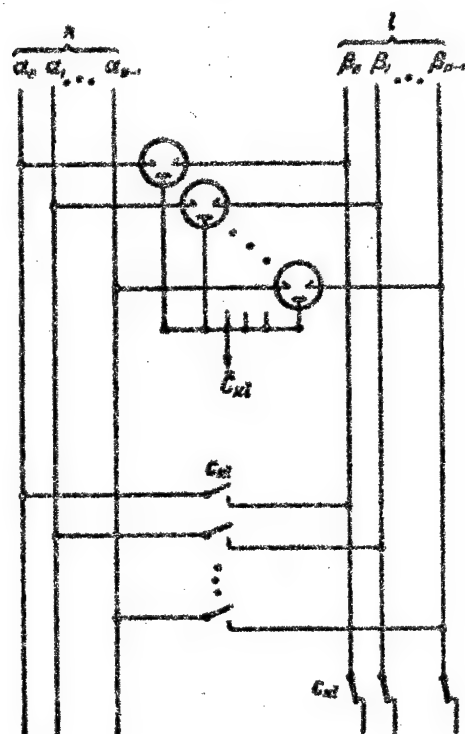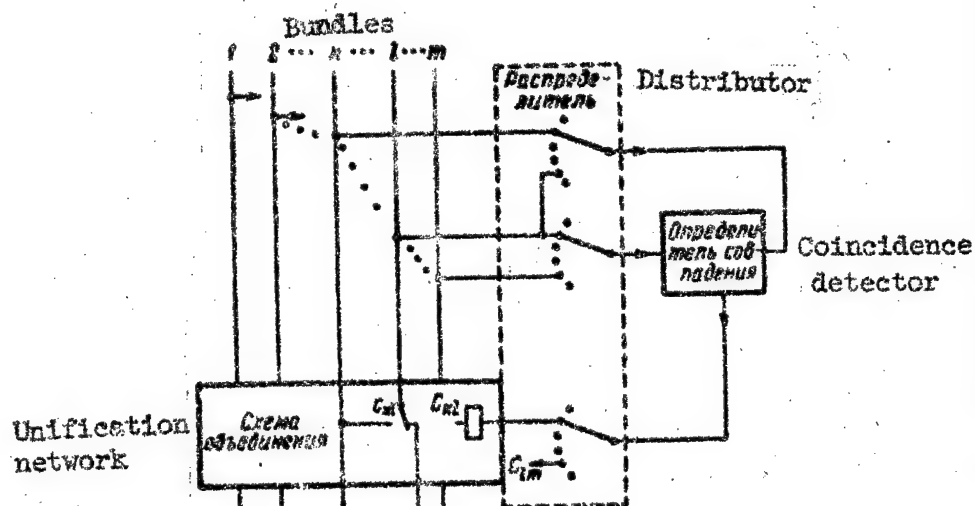
Fig. 102.

Fig. 103

Control of the unification is by means of a unification detector, which is connected by means of a distributor to individual bundles. If a coincidence is established, a suitable unification relay is connected in the detector, as shown in the block diagram of Fig. 103.

When voltages are applied to the wire not through the contacts of a switch, but with the aid of a relay network, the   unification operation can be performed by suitable change in the states of the master relays after briefly unifying the beams once coincidence has been established. The corresponding network is shown in Fig. 104. The master   relays A and B operate in the following manner: when 1 is applied, only relay A operates. When 0 is applied, relay B operates. When $\frac{1}{2}$ is applied, both relays operate. These numbers are applied in the form of brief pulses fed to windings I. If the wires are joined together for a short time, and if the master relay of both bundles are in the same state, their states will not change; but if only one
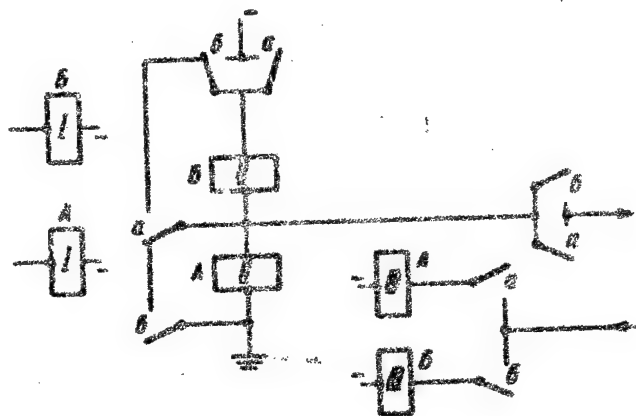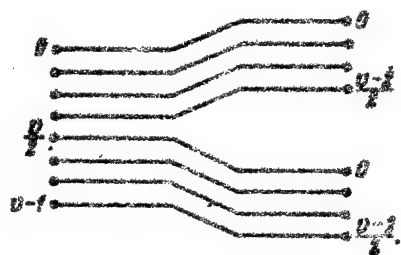
Fig. 104

Fig. 105

relay operates in one bundle, and both **relays operate**
in the second, then the relay operating in the second is
the one operating also in the first, i.e., a change in
potential will take place in the wires, in accordance
with Table 23. The master relays of one of the bundles
should then release, owing to the application of a short
current pulse to windings III.

In the second method there is no need for using a
unification relay at each point.

4. Separation of Assemblies

The separation of numbers into two assemblies
at the point where there is connected a transfer contact
of a relay with weight q is realized in this method of
simulation of assemblies by simply dividing the
bundle into two parts with corresponding renumbering
of the wires in one of them, as shown in Fig. 105.

3. REPLACEMENT OF A CONTACT WITH A DIRECT LEAD

One of the contacts of the transfer group can be

replaced with a direct lead (Chapter 7, Section 4) in
the case when the obligatory numbers of the assembly
of this contact are completely contained in the assembly
of the second contact, even in the form of conditional
numbers. In other words, in this case the following
inequality should be satisfied

$$\{N_1\} < \{N_2, M_2\} \qquad (11.5)$$

or

$$f_{1\,min} \leqslant f_{2\,max}, \qquad (11.6)$$

where $f_{1min} = \{N_1\}$ -- obligatory assembly of the contact,
which can be replaced by a direct lead;

$f_{2max} = \{N_2, M_2\}$ -- assembly of second contact.

Changing over to voltages α and β on the wires
i of bundles A and B, corresponding to make and break
contacts of one group, inequality (11.6), which indicates
the possibility of replacing the make contact (bundle A)

with a direct lead will be satisfied if for each pair of
like wires the function s^i_{AB}, determined by Table 24,
is equal to unity. Analogously, if the function s^i_{BA}
(Table 25) is equal to unity for all pairs of like wires,
this will indicate the possibility of replacing the
break contact (bundle C) with a direct lead.

<div align="center">

Table 24

β_i	α_i		
	0	1/2	1
0	1	1	0
1/2	1	1	1
1	1	1	1

Table 25

β_i	α_i		
	0	1/2	1
0	1	1	1
1/2	1	1	1
1	0	1	1

</div>

The simulation of these functions is analogous
to the simulation of the correspondence function by
double verification with allowance for polarity.
When unifying bundles in which at least one is connected
to the direct lead, a check must be made on the possi-
bility of appearance of false circuits (see Chapter 7,
Section 5).

4.   SYNTHESIS OF NETWORK

To obtain the   synthesized network the machine is provided with a handle, which represents a universal $(1,k)$-pole, in which each contact and connection between wires are indicated by a bulb, as shown in Fig. 106. Bulbs A_1 and A_2 indicate the presence of a given contact in the network and light when there is at least one total voltage in the bundle corresponding to the given output.

Bulbs B_1 and B_2 light when a given contact is eliminated (in which case bulbs A_1 and A_2 must not glow) or is replaced by a direct lead.

Bulbs $C_{k\ell}$ light up in the case when correspondence is established between bundles k and ℓ.

Investigations have shown that the   demi-panel is best constructed in the form of pyramids with vertices at the point of output and a universal network at the input, as shown schematically in Fig. 107. For the universal network the bulbs D (see Fig. 106) are placed at   the intersections of the outputs of both parts of the panel.
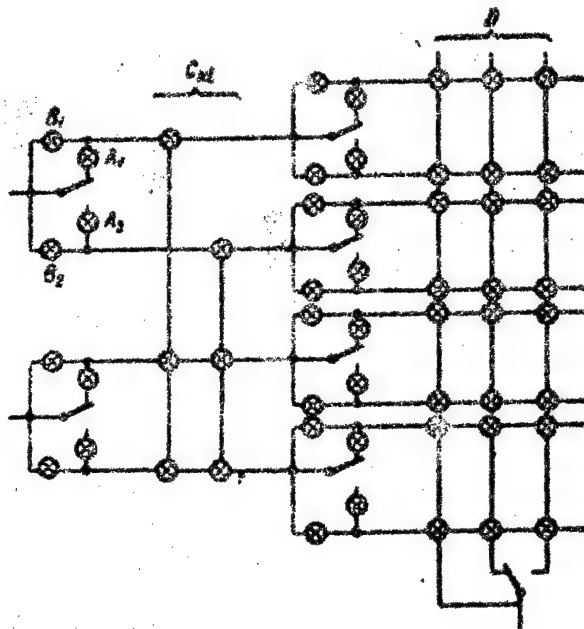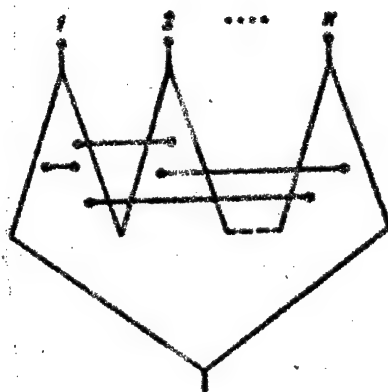
Fig. 106

Fig. 107

5. CHANGE OF BASE AND CHOICE OF VERSION

As already indicated (see Chapter 7, Section 9) the complexity of a network depends on the system used to number the relays, i.e., on the base adopted. A change in base leads to a change in the numbers. In the machine, the changing of the base is simulated by means of a switchboard that changes the numbers of the assemblies in accordance with the change of the numbering of the relays. Such a switchboard has $n!$ states and $2^n - 2$ inputs and outputs, while the numbers 0 and $(2^n - 1)$ do not change when the base is changed.

(The theory of the construction of such switchboards was developed by V. G. Lazarev and Yu. L. Sagalovich /133/.)

After each switching, the machine constructs a new network. The choice of the version is based on certain additional conditions -- the minimum of total number of contacts in the network, a specified distribution of contacts to the relay, etc.

To facilitate the choice, the machine is equipped

with counters for the numbers of contacts on each relay and for the total number of contacts. The counters may be interconnected with the control assembly in such a way that the network is chosen automatically.

6. AUTOMATIZATION OF THE SPECIFICATION OF THE CONDITIONS

For synthesis of sequential networks, the conditions can be specified in the form of connection table in which keys specify the states of the receiving elements and actuating circuits. The machine verifies automatically the realizability of the table, determines the periods during which the states of the intermediate relays must change, and introduces intermediate relays in accordance with a specified program.

After the realizability of the table is established, ~~for each of the actuating circuits~~ the corresponding assemblies are determined for each of the actuating circuits and also for the intermediate-relay circuits. The apparatus for specifying the

connection tables was developed under the leadership of V. G. Lazarev.

7. BLOCK DIAGRAM OF THE MACHINE

Fig. 108 shows a block diagram of the machine for the synthesis of relay networks. The principal blocks of the machine are as follows:

A panel for specifying the connection table with 2-position keys K_1 for the receiving elements, lamps L for the intermediate relays, and 3-position keys K_2 for establishing the states of the actuating circuits in each step.

Base switchboard (BS).

Coincidence detector (CD).

Dummy panel.

Number of contact counters (C).

Control network (Y).

The operation of the machine was verified in a breadboard model constructed in the Laboratory for Wire Communication, Academy of Sciences, U.S.S.R. Later, in

accordance with the model developed in the laboratory, the machine shop of the Institute for Automation and Telemechanics, Academy of Sciences, U.S.S.R., was prepared a model of the machine, which has passed laboratory tests successfully and which was demonstrated at the World's Fair in Brussels in 1958 (it was awarded the highest prize of the fair -- Grand Prix). At the present time work is being done on the development of machines intended for practical utilization in the design of relay networks. (The development was carried out under the leadership of the author with the aid of A. A. Arkhangel'skaya, S. S. Kraynov, V. G. Lazarev, and O. F. Sergeyeva.

A, B, ..., W, X, ..., — relay windings (in multipole-winding relays — windings from which the current flows from the start to the end).

$\overleftarrow{A}, \overleftarrow{B}, ..., \overleftarrow{W}, \overleftarrow{X}, ...$ — bucking windings of relays (current flows from the end to the start).

a, b, ..., w, x — make contacts, Boolean variables.

$\overline{a}, \overline{b}, ..., \overline{w}, \overline{x}, ...$ — break contacts, inverse Boolean variables.

G — element of finite conductivity.

$G_{\simeq A}$ — ditto of same order of conductance as element A.

$G_{>A}$ — ditto, of greater order.

$G_{\geqslant A}$ — ditto of greater order or of the same order.

$G_{<A}$ — ditto of same order.

$G_{\leqslant A}$ — ditto, of smaller or same order.

f, ϕ — contact circuit (its structural formula).

f_W — contact circuit of relay W (operating formula of relay W).

ω — any indeterminate contact circuit.

f_{ij} — contact circuit between nodes i and j.

f_{min} — minimal contact circuit.

f_{max} — maximal contact circuit;

r_W — formula for operation of relay W.

g_W — formula for holding relay W.

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h_W — formula for release of relay W.

k_α — constituent with number .

r — subconstituent, corresponding to the operating step.

g — the same, for holding step.

h — the same, for release step.

α, β, ..., — numbers of states (constituents) of a network

η_i — the same, obligatory constituents.

μ_i — the same, conditional constituents.

ν_i — forbidden numbers.

$$\{\eta_1, \eta_2, \cdots \eta_r (\mu_1, \mu_2, \cdots \mu_s)\} = \{N, M\}$$

— assembly of obligatory and conditional numbers.

N — assembly of obligatory numbers.

M — assembly of conditional numbers.

f_1/f_2 — equivalence.

F, $\emptyset$ — relay circuit (relay network).

Ω — any (indeterminate) relay circuit.

n — number of relays (elements) in a network.

$q = 2^{i-1}$ — weight of relay X_i;

+ — parallel connection (logical addition).

. — series connection (logical multiplication).

0 — permanently opened circuit.

1 — permanently closed circuit.

$\bigcirc$ — joining

= — equivalence of circuits based on structural admittance.

$\doteqdot$ —— equality of relay circuits in action.

$\equiv$ —— absolute equivalence of relay networks.

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